# A NEW DRIVE CONCEPT FOR HIGH-SPEED POSITIONING OF PIEZOELECTRIC ACTUATORS

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**Abstract**
This paper is devoted to the infinite-dimensional control design for a composite piezoelectric trimorph cantilever with complex hysteresis nonlinearities and dynamic creep processes. The control concept being proposed comprises a flatness-based trajectory planning in combination with a passivity-based controller which guarantees the stability of the resulting closed-loop error system. It is well known that at higher electric field strengths the polarization of the piezoelectric material saturates and significant complex hysteretic nonlinearities and dynamic creep effects appear. The mathematical model of the piezoelectric cantilever is approximated in form of a Hammerstein-like model with the hysteretic nonlinearity and the creep dynamic at the input connected in series with a linear infinite-dimensional beam model. The difference principle realized in the trimorph configuration of the piezoelectric bender leads to a special symmetry property of the resulting input nonlinearity and thus admits the application of the Prandtl-Ishlinskii theory for the systematic calculation of an inverse operator for compensating the hysteresis and creep effects. Measurement results on a commercially available serial trimorph bender shows the feasibility of the proposed control strategy, in particular for the case of large displacements.

## Introduction

Piezoelectric Trimorph-bending actuators consist of a substrate of metal or carbon fibre and two metalised piezoceramic films [1]. In the case of a serial trimorph configuration as shown in Fig. 1 the inner electrodes have constant potential (normally ground). The outer electrodes are driven by the positive control voltages $V_1(t) = V_0 + V(t)$ and $V_2(t) = V_0 - V(t)$ with the constant offset voltage $V_0$ which determines the operating point of the piezoelectric layers. The resulting bending moment at the tip of the cantilever due to the control voltages produces a displacement $u(t,x)$ of the beam. For the sake of convenience the tip position as the control variable will be subsequently denoted by $s(t) = u(t,L)$.

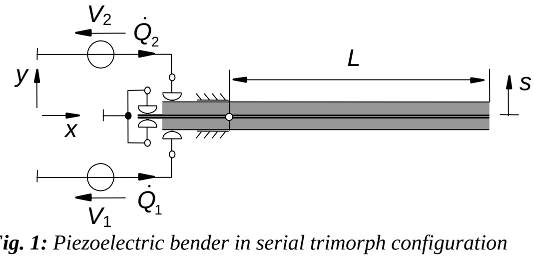


***Fig. 1:*** *Piezoelectric bender in serial trimorph configuration*

Piezoelectric beams are described by partial differential equations and thus are members of the class of infinite-dimensional systems. It is well known that a controller designed on the basis of a finite approximation of the infinite-dimensional model may cause spill-over effects in the closed-loop system. This is why, we will henceforth focus on control strategies which are directly based on the infinite-dimensional model. In recent years there have been some significant advances in extending the concepts of differential flatness and passivity-based control to the infinite-dimensional case. For applications with large displacements the full range of input voltage being available has to be used. For high electric field strengths it is well known that the piezoelectric material shows significant hysteretic and creep behaviour. Furthermore, good progress was made in the modelling and compensation of simultaneously occurring hysteresis and creep effects in piezoelectric layers. All these results can be applied in a beneficial way to the piezoelectric bender under consideration. In the following we will use an approximation in form of a Hammerstein-like model with the ferroelectric hysteretic nonlinearity and the creep dynamic at the input connected in series with a linear infinite-dimensional beam model.

The proposed new control concept shown in Fig. 2 consists of three components: The complex hysteresis and creep effects ($W_P'$) are compensated by an inverse filter ($P_K'^{-1}$) which can be designed by means of the so-called Prandtl-Ishlinskii approach.

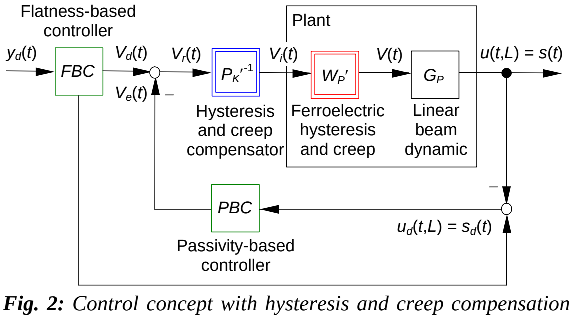


***Fig. 2:*** *Control concept with hysteresis and creep compensation for large displacements*

Based on the infinite-dimensional mathematical model of the piezoelectric bending actuator ($G_P$) a flatness-based open-loop tracking controller (FBC) is derived, such that the effects due to the beam dynamics are suppressed. A passivity-based closed-loop controller (PBC) derived from the infinite-dimensional model ensures that the error system is stabilized.

**Control Concept**

As already mentioned in the introduction, the control concept consists of a flatness-based feedforward controller, see also [4], in combination with a passivity-based closed-loop controller for the resulting error system. For deriving the control concept we assume that the bending actuator, see Fig. 1**,** can be mathematically described by an Euler-Bernoulli beam model. Thereby, the mid-line of the beam is assumed to coincide with the $x$-axis in the stress-free reference state. The displacement of the mid-line of the beam is denoted by $u(t,x)$. The mathematical model reads as, see [4] for a detailed derivation,

$$\begin{aligned} &\mu\partial_t^2 u(t,x)+\Lambda\partial_x^4 u(t,x)=0\\ &\Lambda\partial_x^2 u(t,L)+\Lambda^V(L)V=0\\ &\Lambda\partial_x^3 u(t,L)=0\ . \end{aligned} \tag{1}$$

Thereby, the constants $\mu$, $\Lambda$, $\Lambda^V(L)$ only depend on geometrical and material parameters. The kinematic boundary conditions of the cantilever at the fixed end $x = 0$ are given by $\partial_x u(t,0) = 0$ and $u(t,0) = 0$.

The flatness-based trajectory planning is based on the solution of (1) in the Laplacian domain that yields

$$\hat{u}(x)=\hat{\chi}_1\hat{C}_1(x)+\hat{\chi}_2\hat{S}_1(x)+\hat{\chi}_3\hat{C}_2(x)+\hat{\chi}_4\hat{S}_2(x) \tag{2}$$

with $\hat{p}=\sqrt{Is}(\mu/\Lambda)^{1/4}$, $I=\sqrt{-1}$, the Laplace variable $s$ and the operator functions

$$\begin{aligned} &\hat{C}_1(x)=\left(\cosh(\hat{p}x)+\cos(\hat{p}x)\right)/2\\ &\hat{C}_2(x)=\left(\cosh(\hat{p}x)-\cos(\hat{p}x)\right)/(2\hat{p}^2)\\ &\hat{S}_1(x)=\left(\sinh(\hat{p}x)+\sin(\hat{p}x)\right)/(2\hat{p})\\ &\hat{S}_2(x)=\left(\sinh(\hat{p}x)-\sin(\hat{p}x)\right)/(2\hat{p}^3)\ . \end{aligned} \tag{3}$$

From the kinematic boundary conditions we can immediately deduce that $\hat{\chi}_1=\hat{\chi}_2=0$. Furthermore, by choosing

$$\hat{\chi}_3=-2\frac{\Lambda^V(L)}{\Lambda}\hat{C}_1(L)\hat{y},\qquad \hat{\chi}_4=2\frac{\Lambda^V(L)}{\Lambda}\hat{p}^4\hat{S}_2(L)\hat{y}$$

the dynamic boundary conditions are satisfied for the control input

$$\hat{V}=2\left(\hat{C}_1(L)\hat{C}_1(L)-\hat{p}^4\hat{S}_2(L)\hat{S}_1(L)\right)\hat{y}\ . \tag{4}$$

Thereby, $\hat{y}$ serves as a so-called possible flat output. Furthermore, the bending deflection in operator form $\hat{u}(x)$ can be expressed in terms of the flat output $\hat{y}$ in the form

$$\hat{u}(x)=\Lambda^V(L)\frac{2}{\Lambda}\left[\hat{p}^4\hat{S}_2(L)\hat{S}_2(x)-\hat{C}_1(L)\hat{C}_2(x)\right]\hat{y}.$$

Taking advantage of the power series representation of the operator functions, see [4], [8], we are able to transform the control input $\hat{V}$ and the bending deflection $\hat{u}(x)$ to the time domain by replacing the operator $s^k$ with $\mathrm{d}^k/\mathrm{d}t^k$, $k=1,2,\ldots$. Since this task is straightforward we will only perform this transformation for $\hat{V}$. By means of the power series representation for $\hat{C}_1(x)$, see [4], the control input in the time domain reads as

$$V(t)=y(t)+\sum_{k=0}^{\infty}\frac{\left(4L^4\mu/\Lambda\right)^k}{(4k)!}\frac{\mathrm{d}^{2k}}{\mathrm{d}t^{2k}}y(t)\ . \tag{5}$$

For solving the trajectory planning problem a desired flat output $y_d(t)$ has to be specified and from this the associated control input $V_d(t)$ and the beam deflection $u_d(t,x)$ can be calculated.

In the next step of the controller design task we will exploit the concept of passivity to stabilize the infinite-dimensional error system. Introducing the deflection error $u_e(t,x) = u(t,x) - u_d(t,x)$ and the additional control input $V_e(t) = V(t) - V_d(t)$, we get, according to the linearity of (1), the error system in the form

$$\begin{aligned} &\mu\partial_t^2 u_e(t,x)+\Lambda\partial_x^4 u_e(t,x)=0\\ &\Lambda\partial_x^2 u_e(t,L)+\Lambda^V(L)V=0,\ \Lambda\partial_x^3 u_e(t,L)=0\\ &u_e(t,0)=\partial_x u_e(t,0)=0\ . \end{aligned} \tag{6}$$

The Hamiltonian of the free error system, i.e. for $V_e = 0$, is given by the expression [4]

$$H_0=\frac{1}{2}\int_0^L \Lambda\left(\partial_x^2 u_e\right)^2+\mu\left(\partial_t u_e\right)^2\mathrm{d}x\ . \tag{7}$$

Then, by making use of the integration by parts technique the change of $H_0$ along a trajectory of the error system results in

$$\frac{\mathrm{d}}{\mathrm{d}t}H_0=-\Lambda^V(L)V_e\,\partial_t\partial_x u_e(L)\ . \tag{8}$$

Thus, the velocity of the tip angle provides a collocated output $y_c = \partial_t\partial_x u_e(L)$ for the control input $V_e$, see, e.g., [6]. At this point it is worth mentioning that the collocated output $y_c$ can also be directly measured by utilizing the direct piezoelectric effect of the piezoelectric layers. On condition that the sensor electrodes are short-circuited by a charge amplifier the time derivative of the electric charge corresponds to the collocated output $y_c$, see, e.g., [6] for further details. As in our case sensor layers are not provided, we can design a modal observer to determine the collocated output $y_c$ by means of measuring the velocity $\mathrm{d}s/\mathrm{d}t$ of the tip position. Now, the simple controller

$$V_e=-ky_c=-k\partial_t\partial_x u_e(L)\quad,\quad k>0 \tag{9}$$

renders (8) negative semi-definite. This gives at least a necessary condition for the asymptotic stability, see [4] and [5] for a more detailed treatment of the stability consideration and an explicit proof even for a larger class of stabilizing controller.

## Complex hysteresis and creep compensation

Neglecting the self-generated electric field due to the piezoelectric effect, we get the linear relation

$$Q_1 = Q_{10} + C_P V \qquad (10)$$

between the voltage $V$ and the charge $Q_1$ for sufficiently small signals. The factor $C_P$ is the small-signal capacity of one piezoelectric layer in the operating point ($V_0$,$Q_{10}$). However, as depicted in Fig. 3, measurements show that for large voltages $V$ this is no longer valid. Therefore, (10) will be replaced by a relation of the form

$$Q_1 = Q_{10} + W_1[V] \qquad (11)$$

with an appropriate hysteresis and creep operator $W_1$. Analogously, for the upper piezoelectric actuator layer the charge $Q_2$ results from

$$Q_2 = Q_{20} + W_2[-V] \qquad (12)$$

and thus the electric charge of the middle electrode in Fig.1 is given by

$$Q = Q_{10} + W_1[V] - (Q_{20} + W_2[-V]) \,. \qquad (13)$$

Due to the fact that the two piezoelectric layers are build up identically and both layers are driven in the same operating point the two nonlinearities can be assumed to be identical, i.e. $W_1 = W_2 = W$. Thus, we have $Q = W_P[V]$ with the symmetry property

$$W_P[V] = W[V] - W[-V] = -W_P[-V] \qquad (14)$$

which is a necessary condition to approximate the nonlinearity $W_P$ by the sum $P_K = P + K$ of a Prandtl-Ishlinskii hysteresis operator $P$ and a Prandtl-Ishlinskii creep operator $K$. This implies that the relation (13) can be replaced by

$$Q = P_K[V] \,. \qquad (15)$$

In practice the charges $Q_1$ and $Q_2$ are measured by Sawyer-Tower circuits, see, e.g., [2] and the charge of the middle electrode is calculated by $Q = Q_1 - Q_2$. With the measurements of $Q$ and $V$ we can directly identify the operator $P_K$ by means of the identification methods described in [7].

Now, (15) motivates to formulate the serial trimorph bender of Fig. 1 in form of a Hammerstein-like model as shown in Fig. 2 with $V = W_P'[V_i] = W_P[V_i]/(2C_P)$ and $V_i = P_K'^{-1}[V_r] = P_K^{-1}[2C_P V_r] = P_K^{-1}[Q_r]$. This structure enables us the compensate the hysteretic and creep nonlinearity at the input by means of a compensator $P_K^{-1}$ satisfying the relation

$$P_K[P_K^{-1}[Q]] = Q \,. \qquad (16)$$

The compensator $P_K^{-1}$, defined by the relation $V = P_K^{-1}[Q]$, can be determined by the following implicit operator equation

$$V = P^{-1}[Q - K[V]] \,. \qquad (17)$$

For further details concerning the theoretical and numerical aspects of the solution of (17) the interested reader is referred to [3]. Fig 3. shows the experimental results obtained for a serial trimorph bender as shown in Fig. 1.

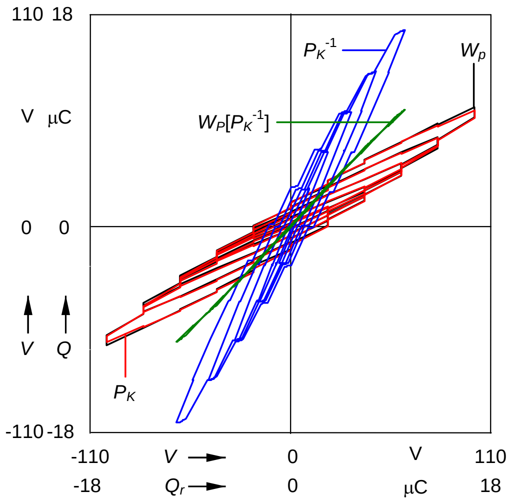


***Fig. 3:*** *Measurement of hysteresis and creep compensation*

It can be seen that the measured hysteresis and creep characteristics $Q = W_P[V]$ due to (14) and Fig. 3 can be accurately approximated by a Prandtl-Ishlinskii hysteresis and creep operator $P_K$. The relative model error defined by

$$e_{P_K} := \frac{\left\| P_K[V] - Q \right\|_\infty}{\left\| Q \right\|_\infty}$$

is about 1.03 %. Furthermore, Fig. 3 depicts the inverse operator $P_K^{-1}$ and the result of the compensation procedure $W_P[P_K^{-1}]$.

## Experimental results

The control concept as depicted in Fig. 2 was implemented in the real-time environment of dSPACE. The piezoelectric actuator under consideration is the serial trimorph bender VIBRIT 1100 of Argillon, see [1]. Fig. 4a depicts the step response of the uncontrolled beam for a step input of the voltage $V(t)$ with an amplitude of 50 V. Due to the small structural damping the step-response of the uncontrolled bending actuator has a large overshoot and a large settling time. Fig. 4b shows the response in the tip position $s$ of the flatness-based open-loop control for a reference trajectory of the tip position $s_d$ with a rising time of 10ms. Especially for small displacements the performance of the controller is very good. Overshoot and settling time of the beam are strongly reduced. For higher electrical voltages needed for larger displacements the influence of hysteresis and creep effects becomes evident. This results in a larger position error $s_d - s$, overshoot and settling time. Fig. 4c depicts the result of the flatness-based open-loop control in combination with the hysteresis and creep compensation for the same reference trajectories as in Fig. 4b. The remaining position error and the overshoot can be strongly reduced. As shown in Fig. 4d a further reduction of the position error can be realised with the proposed passivity-based error feedback controller in Fig.2.

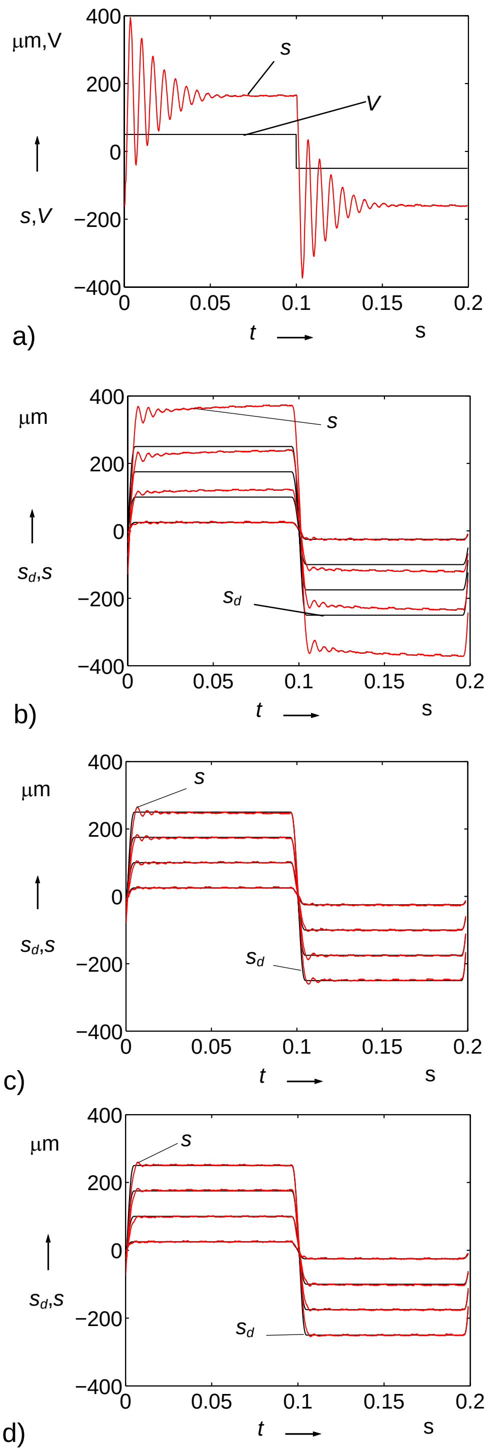


***Fig. 4:*** *Tracking behaviour of the piezoelectric bender: a) Step response b) Open-loop flatness-based controller c) Additional hysteresis and creep compensator d) Additional passivity-based feedback controller*

The result is a high performance tracking behaviour of the piezoelectric bender in the large signal range, important for practical applications.

## Summary

In this paper we have presented a control concept for a composite piezoelectric cantilever with large displacements consisting of a hysteresis and creep compensation based on the Prandtl-Ishlinskii approach, a flatness-based open-loop controller and a passivity-based error feedback controller which guarantees the exponential stability of the overall closed-loop system. The control concept was implemented on an experimental setup for a commercially available serial trimorph bender. The measurement results show the feasibility and high performance of the proposed control design.

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