# Self-Learning Compensation of Hysteretic and Creep Nonlinearities in Piezoelectric Actuators

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**Abstract:**
Solid-state actuators based on active materials allow high operating frequencies with nearly unlimited displacement resolution. This predestines them, for example, for application in highly precise positioning systems. However, the nonlinear behaviour in such systems is mainly attributed to actuator transfer characteristics as a result of driving with high amplitudes. In this paper a novel self-learning compensation method based on the so-called modified Prandtl-Ishlinskii approach is presented, which allows extensive compensation of the complex solid-state actuator hysteretic and creep nonlinearities during operation. Finally, it is shown in an example involving a two-axis piezoelectric parallel-kinematic positioning system that this compensation substantially decreases the deviations of the actual output displacements from the desired displacements of the controlled system.

Keywords: piezoactuator, hysteresis, creep, self-learning compensation, modified Prandtl-Ishlinskii approach, projected dynamical systems, parallel-kinematic positioning system

## Introduction

Solid-state actuators based on active materials allow high operating frequencies with nearly unlimited displacement resolution. This predestines them, for example, for application in highly precise parallel-kinematic positioning systems. Herewith, the end effector is moved by suitable actuators opposite to a fixed frame [1]. The required guiding elements are mainly realised as elastic joints. Thus, hard nonlinearities such as static and sliding friction are not present. Consequently, the mechanics can be regarded as a linear structural dynamic system consisting of spatially distributed masses, dampers and springs. Thus, the nonlinear behaviour in such systems is mainly attributed to actuator transfer characteristics as a result of driving with high amplitudes.

The actuator transfer behaviour is normally of hysteretic nature and contains in addition complex dynamic creep effects, which emerge more or less in dependence of the material used [2], [4]. Therefore, solutions for compensating these undesirable nonlinear effects must be developed. In previous publications [2], [3] an off-line compensator synthesis was introduced which occurs one-time before the start-up of the positioning system. A disadvantage of this approach consists in the fact that influences like temperature variations or aging remain disregarded. To avoid this, the method pursued here is to identify and to model hysteretic and creep nonlinearities in two phases during operation, i.e. on-line, and on this basis in the third step to synthesise a compensator.

In the first phase a time constant data basis for constructing the quadratic cost function from the output-input measurement data is generated. The cost function is minimised in the subsequent learning phase. As a result the model parameters are identified and a compensator is synthesised. The definition of suitable time intervals for the building and learning phases depends on certain requirements (driving signal form, model order) and is discussed in [4]. The superordinate flow control is taken over by a synchronous finite Moore machine. With a cyclic execution of the single phases the compensator nearly reaches its optimum after a few cycles.

In the following modelling the hysteretic and creep actuator nonlinearities is introduced at first with the help of operator calculus. The theory of projected dynamical systems allows model identification and compensator synthesis during operation. Finally, the efficiency of the developed method is verified in an example involving a two-axis piezoelectric parallel-kinematic positioning system and it is shown that on-line compensation of nonlinearities substantially decreases the deviations of the actual output displacements from the desired displacements of the controlled system.

## Concept of self-learning compensator

The signal flow diagram in Fig. 1 forms the basis of realising the self-learning compensator of complex actuator hysteretic and creep nonlinearities. Here

$x(t)$ and $y(t)$ are the input and output signals of the actuator; $y_r(t)$ is a given control signal. In the first step follows the mathematical description of the real output-input actuator characteristic using hysteresis operators. Suitable operators include the Krasnosel'skii-Pokrovskii operator [5] and the Preisach operator [6]. After identifying the model parameters during actuator operation (see *Experimental results*) the compensator synthesis occurs.

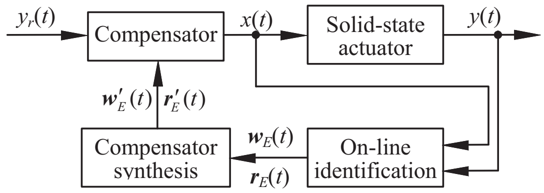


***Fig. 1:*** *Signal flow diagram of self-learning compensator*

The inverse model required for compensating the actuator nonlinearities is calculated numerically. In order to reduce the high computational demand, the so-called Prandtl-Ishlinskii hysteresis (PIH) operator is recommended here. It belongs to the class of Preisach operators and guarantees a direct synthesis of the compensator, i.e. its inversion can be carried out analytically, and the inverse operator exhibits the same structure as the original operator. On this account the PIH operator is very advantageous for real-time applications.

However, this operator exhibits point-symmetry about the origin [3]. If an unsymmetrical actuator transfer behaviour is present (for example, due to an unsymmetrical driving signal with respect to the zero point or as a result of saturation effects at high input signal amplitudes), one can unsymmetrically deform the point-symmetrical PIH operator by means of the subsequent nonlinear and memoryless Prandtl-Ishlinskii superposition (PIS) operator. A memoryless nonlinearity is understood as a non-ambiguous characteristic behaviour which exhibits no branchings [3].

To consider in the modelling the complex dynamic creep effects appearing in piezoactuators, the PIH operator is complemented here with a creep operator.

### A. Modelling of complex hysteresis and creep

One obtains the complex operators for representing the real phenomena by the weighted linear superposition of infinitely many elementary operators with different characteristic parameters as so-called threshold-continuous model. A finite-dimensional, threshold-discrete approximation of the threshold-continuous model is necessary for application in control and signal processing algorithms. The so-called threshold-discrete modified Prandtl-Ishlinskii hysteresis operator, for example, is defined by

$$
\begin{aligned}
M_\delta[x](t) &= S_\delta\left[H_\delta[x]\right](t) \\
&= \sum_{i=-m}^{m} w_{Si} S_{r_{Si}}\left[\sum_{i=0}^{n} w_{Hi} H_{r_{Hi}}\left[x, z_{H0i}\right]\right](t),
\end{aligned}
\tag{1}
$$

through the serial connection of the PIH operator $H_\delta$ and the PIS operator $S_\delta$ to form a complete model for representing the unsymmetrical hysteretic behaviour. The threshold values $r_{Hi}$, $i = 0\ldots n$, and $r_{Si}$, $i = -m\ldots m$ are the characteristic parameters of the PIH and PIS operators, respectively [3].

To be able to model the complex creep effects depending on the prehistory of the input signal $x$, the Prandtl-Ishlinskii creep (PIC) operator $K_\delta$ is introduced, whose order is selected equal to the order $n$ of the PIH operator [2]. The PIC operator is connected in the hysteretic model (1) in parallel with the PIH operator $H_\delta$ and one obtains, finally, as an entire model [2],

$$
\begin{aligned}
M_{K\delta}[x](t) &= S_\delta\left[H_\delta[x] + K_\delta[x]\right](t) \\
&= \sum_{i=-m}^{m} w_{Si} S_{r_{Si}}\left[\sum_{i=0}^{n} w_{Hi} H_{r_{Hi}}\left[x, z_{H0i}\right] + \right. \\
&\qquad \left. + \sum_{i=0}^{n} w_{Ki} K_{r_{Ki}}\left[x, z_{K0i}\right]\right](t).
\end{aligned}
\tag{2}
$$

### B. Identifying and inverting

For describing the deviation of the model behaviour $\boldsymbol{w}_E^T \boldsymbol{\Xi}_E[x](t)$ from the measured behaviour of the real system $y(t)$ for every time $t$ a scalar output error model

$$E[x](t, \boldsymbol{w}_E) = \boldsymbol{w}_E^T \boldsymbol{\Xi}_E[x](t) - y(t) \tag{3}$$

is defined, which linearly depends on the model parameters $\boldsymbol{w}_E \in \Re^d$ to be identified. The variable $d$ indicates the parameter space dimension. $\boldsymbol{\Xi}_E[x](t)$ is a vector time function depending on the input signal and represents the model behaviour.

Additionally, a scalar cost function with the start time $t_0$ and end time $t_E$ is determined

$$V(\boldsymbol{w}_E) = \frac{1}{2}\int_{t_0}^{t_E} E^2[x](t, \boldsymbol{w}_E)\,\mathrm{d}t, \tag{4}$$

which provides a quadratic measure for the deviation of the model behaviour from the system behaviour. The purpose of the identification is to minimise the cost function $V$ depending on the model parameters $\boldsymbol{w}_E$. The existence of a global minimum can only be guaranteed in the case of a linear or convex cost function [4].

Due to the structure of the entire model (2) the error model (3) and the cost function (4) are nonlinearly dependent on the model parameters $w_{Hi}$ and $w_{Ki}$, $i = 0\ldots n$. To ensure linearity, the error model

$$E[x,y](t,\boldsymbol{w}_E) = H_\delta[x](t) - S_\delta^{-1}[y](t) + K_\delta[x](t) = \boldsymbol{w}_E^T \hat{\boldsymbol{\Xi}}_E[x,y](t) + x(t) \quad (5)$$

with

$$\boldsymbol{w}_E^T = \left(w_{H1}\ \ldots\ w_{Hn}\ w'_{S-m}\ \ldots\ w'_{Sm}\ w_{K0}\ \ldots\ w_{Kn}\right),$$

$$\hat{\boldsymbol{\Xi}}_E^T[x,y](t) = \left(H_{r_{H1}}\ \ldots\ H_{r_{Hn}}\ \text{-}S_{r'_{S-m}}\ \ldots\ \text{-}S_{r'_{Sm}}\ K_{r_{K0}}\ \ldots\ K_{r_{Kn}}\right)$$

is suggested just as in [2], [4]. By means of the equation (5) the inverse PIS operator $S_\delta^{-1}$ is identified. Due to the property of the Prandtl-Ishlinskii method the operator $S_\delta$ can be determined analytically from $S_\delta^{-1}$ [3].

The main condition for inverting the entire model (2) is the continuity of the elementary operators and the strict monotony of all possible branchings. This is fulfilled if the sum of the weights $w_{Hi}$, $i = 0\ldots n$, as well as the sum of the weights $w_{Si}$, $i = -m\ldots 0$ and $i = 0\ldots m$, is positive definite. To ensure thermodynamic consistency the following must apply as well: $w_{Hi} \geq 0,\ w_{Ki} \geq 0,\ i = 0...n$ [3], [4]. By virtue of these inequality constrains the admissible parameter set $K_E \subset \mathfrak{R}^d$, where the weights are identified, is constrained. Therefore, the cost function (4) should be minimised by means of the so-called quadratic program [2], [4]

$$\boldsymbol{w}_E^{\#} = \arg\min_{\boldsymbol{w}_E \in K_E} \left\{V\left(\boldsymbol{w}_E\right)\right\}. \quad (6)$$

Since the model and compensator synthesis occur during operation, the computational demand necessary to solve the numerical solution to the quadratic program results in higher sampling periods of the time-discrete control. For this reason, an alternative method is suggested here based on the theory of projected dynamical systems [7]. The starting point is to construct a differential equation in which stable equilibrium point $\boldsymbol{w}_{E\infty}$ matches the solution of the optimisation problem (6). It is called the vector differential equation with projected gradient vector field and is defined by

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{w}_E(t) = \boldsymbol{Q}\left(\boldsymbol{w}_E(t), -\nabla V\left(\boldsymbol{w}_E(t)\right)\right) \quad (7)$$

with $\boldsymbol{w}_E(t_0) = \boldsymbol{w}_{E0} \in K_E$ [4], [7]. The projection mapping $\boldsymbol{Q}$ is determined as the orthogonal projection of $\mathfrak{R}^d$ onto the admissible parameter set $K_E$. Consequently, the solution trajectory (a) starting from the initial value $\boldsymbol{w}_{E0} \in K_E$ and leaving the parameter set $K_E$ is displaced to the boundary $\partial K_E$, see Fig. 2 with $d = 2$.

In the following, the differential equation (7) is transformed into a difference equation and solved on-line during operation after the data basis from the output-input measurement data was generated. As a result, the model parameter set is obtained, which is necessary for compensator synthesis. For lack of space, the details concerning the numerical solvability of the difference equation as well as the existence of the solution trajectory, its uniqueness and its global exponential stability are referred to the literature [4], [7].

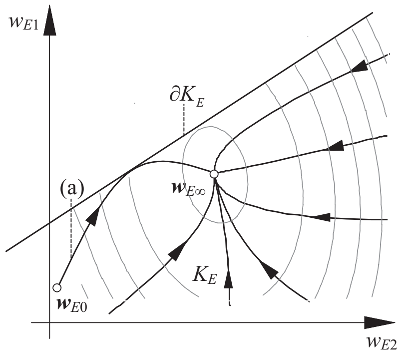


***Fig. 2:*** *Geometrical interpretation of equation (7)*

Thus, the compensator is explicitly determined as the inverse model to (2):

$$M_{K\delta}^{-1}[y](t) = H_\delta^{-1}\left[S_\delta^{-1}[y] - K_\delta[x]\right](t) = \sum_{i=0}^{n} w'_{Hi} H_{r'_{Hi}}\left[\sum_{i=-m}^{m} w'_{Si} S_{r'_{Si}}[y] - \sum_{i=0}^{n} w_{Ki} K_{r_{Ki}}\left[x, z_{K0i}\right], z'_{H0i}\right](t). \quad (8)$$

The inverse PIS operator $S_\delta^{-1}$ is identified by means of the error model (5), and the inverse PIH operator $H_\delta^{-1}$ can be determined analytically from $H_\delta$ [3].

**Experimental results**

In two-axis positioning system shown in Fig. 3 the two opposing piezoactuators can be driven in push-pull operation which leads to a point-symmetrical hysteresis loop allowing modelling by means of the PIH operator. In the following, however, every axis is driven by only one actuator, in order to demonstrate the efficiency of the modified PI method.

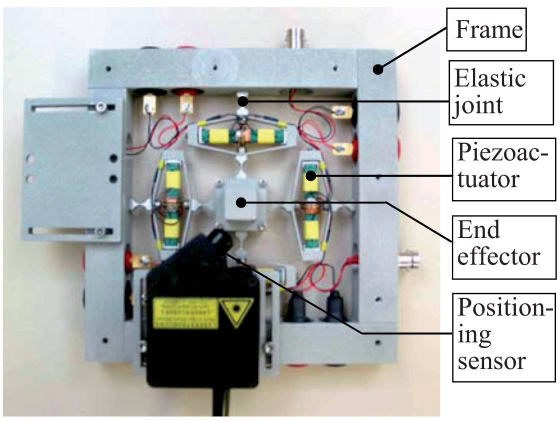


***Fig. 3:*** *Piezoelectric positioning system*

In the first step, the model orders of each single operator are determined: $n = 5$, $m = 3$. This is a compromise between model accuracy and model complexity. The threshold values $\boldsymbol{r}_E^T = \left(\boldsymbol{r}_H^T\ \boldsymbol{r}_S'^T\ \boldsymbol{r}_K^T\right)$, see Fig. 1, are equidistantly distributed above the amplitude range of the driving signal and the output signal of the actuator. After a reset phase of 0.1 s data is acquired within a duration of $T_b = 8$ s. Once the data basis is created the learning phase with $T_l = 1.9$ s follows, providing the basis for the identification of the model parameter set $\boldsymbol{w}_E^T = \left(\boldsymbol{w}_H^T\ \boldsymbol{w}_S'^T\ \boldsymbol{w}_K^T\right)$ see Fig. 1. According to the error model (5) the inverse PIS operator is identified here; its parameters are indicated with an apostrophe. In the last step the compensator parameters $\boldsymbol{r}_E'^T = \left(\boldsymbol{r}_H'^T\ \boldsymbol{r}_S'^T\ \boldsymbol{r}_K^T\right)$ and $\boldsymbol{w}_E'^T = \left(\boldsymbol{w}_H'^T\ \boldsymbol{w}_S'^T\ \boldsymbol{w}_K^T\right)$ are ascertained.

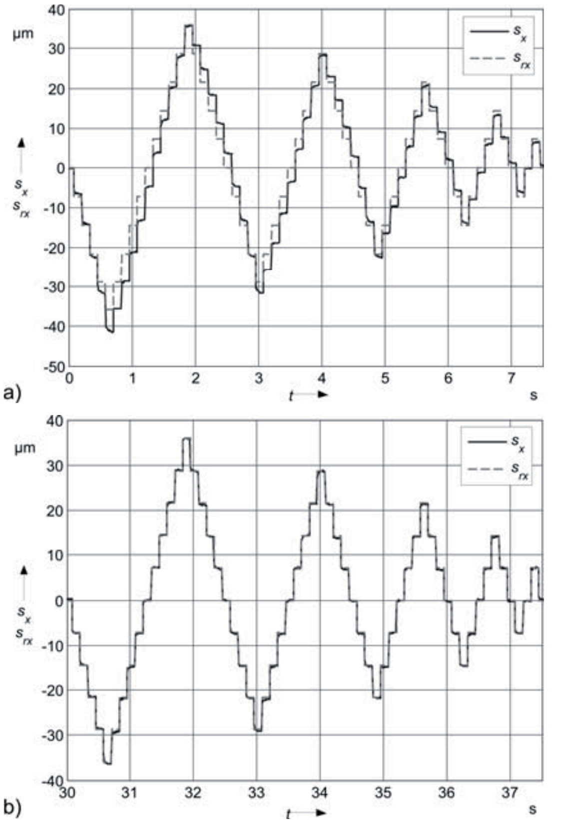


*Fig. 4: Time plots of the actual value $s_x(t)$ and the reference value $s_{rx}(t)$ of a positioning system: a) without compensation, b) with compensation*

The durations of each phase are determined according to the following aspects. The reset phase serves to delete the data basis and, therefore, has to be preferably short. During the building phase it is necessary to ensure that all threshold values of the model are sufficiently excited. The duration of $T_b = 8$ s meets this requirement. The duration of the learning phase $T_l$ is the multiple of the sampling period (125 μs), thus, ensuring a stable solution of projected difference equation [4]. The single phases are integrated into cyclic operation by which the compensator nearly reaches its optimum after a few cycles.

Fig. 4 shows as an example the measured staircase-shaped time plots of the actual output displacement $s_x$ and of the reference displacement $s_{rx}$ of the positioning system. Since the $y$-axis shows similar behaviour, the corresponding demonstration is not necessary.

In Fig. 4a it is depicted how the actual displacements deviate from the reference displacements due to the actuator nonlinearities. The self-learning compensation method affects sufficient compensation of the nonlinearities (Fig. 4b) so that the actual positioning values can correspond to the reference positioning values with a high degree of accuracy.

## Outlook

The research work introduced in this article will be further developed concerning adaptive identification and compensation methods for complex hysteretic and creep nonlinearities. In future, identifying the model parameters will be carried out on the basis of continuously measured values of the input-output behaviour; the data basis is persistently updated in real time.

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