Modeling, Identification and Compensation of Complex Hysteretic Nonlinearities

A modified Prandtl-Ishlinskii Approach

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Abstract

Undesired complex hysteretic nonlinearities are present to varying degree in virtually all smart material based sensors and actuators provided that they are driven with sufficiently high amplitudes. This necessitates the development of purely phenomenological models which characterize these nonlinearities in a way which is sufficiently accurate, robust, amenable to control design for nonlinearity compensation and efficient enough for use in real-time applications. To fulfill these demanding requirements the present paper describes a new compensator design method for invertible complex hysteretic nonlinearities which is based on the so-called modified Prandtl-Ishlinskii hysteresis operator. The parameter identification of this model can be formulated as a quadratic optimization problem which produces the best L_2^2-norm approximation for the measured input-output data of the real hysteretic nonlinearity. Special linear inequality and equality constraints for the parameters guarantee the unique solvability of the identification problem and the invertibility of the identified model. This leads to a robustness of the identification procedure against unknown measurement errors, unknown model errors and unknown model orders. The corresponding compensator can be directly calculated and thus efficiently implemented from the model by analytical transformation laws. Finally the compensator design method is used to generate an inverse feedforward controller for a magnetostrictive actuator. In comparision to the conventional controlled magnetostrictive actuator the nonlinearity error of the inverse controlled magnetostrictive actuator is lowered from about 50 % to about 3 %.

Keywords

Hysteresis, Nonlinear Systems, Modeling, Identification, Hysteresis Compensation

1 Introduction

Complex memory-free nonlinearities or in generalization complex hysteretic nonlinearities are present to varying degree in virtually all smart material based sensors and actuators provided that they are driven with sufficiently high amplitudes. Well known examples for complex hysteretic nonlinearities in smart material systems are piezoelectric, magnetostrictive and shape memory alloy based actuators and sensors [2]. In many applications, these nonlinearities can be limited through the choice of proper materials and operating regimes so that linear sensor and actuator characteristics can be assumed. In the consequence of more stringent performance requirements a large number of systems are currently operated in regimes in which memory-free or hysteretic nonlinearities are unavoidable. This necessitates the development of purely phenomenological models which characterize these nonlinearities in a way which is sufficiently accurate, robust, amenable to control design for nonlinearity compensation and efficient enough for use in real-time applications.

Models of hysteretic nonlinearities have evolved from two different branches of physics: ferromagnetism and plasticity theory. The roots of both branches go back to the end of the 19th century. But only at the beginning of the seventies of the 20th century a mathematical formalism for a systematic consideration of hysteretic nonlinearities was developed. The core of this theory is formed by so-called hysteresis operators which describe hysteretic transducers as a mapping between function spaces [7]. But it is only since the beginning of the 90s that engineers employ this theory on a larger scale to develop modern strategies for the linearisation of hysteretic systems with an inverse control approach. One reason for this is the increasing number of mechatronic applications in recent years which use new solid-state actuators based on magnetostrictive or piezoelectric material or shape memory alloys. Whereas in the beginning mainly the well-known Preisach operator was used for the modeling and linearization of solid-state actuators with the inverse control approach [5,12], recent papers also reference the so-called Krasnosel'skii-Prokrovskii operator [15] and the so-called Prandtl-Ishlinskii operator [4,6,9] which belongs to an important subclass of the Preisach operator [3]. In contrast to the general Preisach and Krasnosel'skii-Prokrovskii hysteresis modelling approach this subclass permits to design the compensator analytically which is an excellent precondition for the use in real-time applications [3,8]. Other well-known hysteresis compensation techniques use compensators based on models for so-called local hysteretic nonlinearities [1,13]. But these models are to simple in its memory structure to model the complex hysteresis nonlinearities in smart material systems in a sufficiently precise way.

To develop a consistent phenomenological design concept for a compensator of an invertible complex hysteretic nonlinearity which is sufficiently flexible in its modeling capabilities and moreover well-suited for real-time applications is not a simple task because it covers in general the following coupled design steps: modeling the real hysteretic nonlinearity, identification of the model parameters to adapt the model to the real hysteretic nonlinearity and inversion of the model to obtain the desired compensator. Especially the mathematical complexity of the identification and inversion problem depends on the phenomenological modeling method (for example Preisach, Krasnosel'skii-Prokrovskii or Prandtl-Ishlinskii modeling) and influences strongly the practical use of the design concept. Another difficulty of the identification problem follows from the strong sensitivity of the model parameters to unknown measurement errors of the input-output data, unknown model errors and unknown model orders. Due to these effects a parameter identification can result in the best case to a poor model accuracy or in the worst case to a locally non-invertible model and as a consequence the whole compensator design fails. Therefore the robustness against these effects is an inherent requirement for a consistent phenomenological compensator design method.

To overcome these difficulties the present paper describes a new compensator design concept for complex hysteretic nonlinearities based on the Prandtl-Ishlinskii modeling approach which is robust in the sense mentioned above. The robustness of the new compensator design method is reached by the consideration of linear inequality constraints for the free model parameters which guarantee a search for the best L_2^2-norm approximation of the measured output-input data only in those parameter ranges where the identified model is invertible.

2 Prandtl-Ishlinskii Modeling and Compensation of Complex Hysteretic Nonlinearities

In the mathematical literature the notation of the hysteretic nonlinearity will be equated with the notation "rate-independent memory effect" [3,10,14]. This means that the present output signal value of a system with hysteresis depends not only on the present value of the input signal but also on the order of their amplitudes, especially their extremum values, but not on their rate in the past.

At the beginning of the 20th century Madelung investigated experimentally the branchings and loopings of ferromagnetic hysteresis which result from the rate-independent memory property and stated the following three rules from his observations [3], see Fig. 1.

1. Any curve C_1 emanating from a turning point A of the output-input trajectory is uniquely determined by the coordinates of A.
2. If any point B on the curve C_1 becomes a new turning point, then the curve C_2 originating at B leads back to the point A.
3. If the curve C_2 is continued beyond the point A, then it coincides with the continuation of the curve C which led to the point A before the C_1-C_2 - cycle was traversed.

Additionally to these three Madelung's rules a fourth important observation can be made for actuator and sensor characteristics of smart materials, and it is exactly this property of real hysteretic nonlinearities in which the complex ones differ from the non complex ones.

4. From a non turning point D within the hysteretic region Ω more than one branch can be traversed.

The branch which was traversed is uniquely determined by the relevant past history of the input signal.

2.1 Modeling of Complex Hysteretic Nonlinearities

Because of its phenomenological character the concept of hysteresis operators developed by Krasnosel'skii and Pokrovskii in the 1970s allows a powerful modeling of complex hysteretic nonlinearities [7]. The basic idea consists of the modeling of the real complex hysteretic nonlinearities by the weighted superposition of many so-called elementary hysteresis operators. One of the most familiar and most important elementary hysteretic mapping

$$y(t) = H_{r_H}[x, y_0](t) \tag{1}$$

between the input signal x and the output signal y is the so-called play or backlash operator H_{r_H} which is often used to model mechanical play in gears with one degree of freedom. It is normally defined by the recursive equation

$$y(t) = H(x(t), y(t_i), r_H) \tag{2}$$

with the initial condition

$$y(t_0) = H(x(t_0), y_0, r_H) \tag{3}$$

for the output signal at initial time t_0. It depends on the independent initial value y_0 of the output and the sliding symmetrical dead-zone function

$$H(x, y, r_H) = \max\{x - r_H, \min\{x + r_H, y\}\} \tag{4}$$

for piecewise monotonous input signals with a monotonicity partition $t_0 \leq t_1 \leq . . \leq t_i \leq t \leq t_{i+1}$ . . $\leq t_N = t_e$ [3]. The operator is characterized by its threshold parameter $r_H \in \Re^+_0$. Fig. 2 shows the rate-independent output-input trajectory of this elementary hysteresis operator.

Although the three Madelung's rules hold for the play operator it can be easily realized that the hysteretic sensor and actuator characteristics of real smart materials are of much higher complexity, note also rule 4.

To obtain a more powerful model for hysteretic nonlinearities we introduce the so-called threshold-discrete Prandtl-Ishlinskii hysteresis operator H by the linear weighted superposition of many play operators with different threshold values. From this follows

$$H[x](t) := \boldsymbol{w}_H{}^T \cdot \boldsymbol{H}_{\boldsymbol{r}_H}[x,\boldsymbol{z}_{H0}](t) \tag{5}$$

with the vector of weights $\boldsymbol{w}_H{}^T = (w_{H0}\ w_{H1}\ ..\ w_{Hn})$, the vector of thresholds $\boldsymbol{r}_H{}^T = (r_{H0}\ r_{H1}\ ..\ r_{Hn})$ with $0 = r_{H0} < r_{H1} < .. < r_{Hn} < +\infty$, the vector of initial states $\boldsymbol{z}_{H0}{}^T = (z_{H00}\ z_{H01}\ ..\ z_{H0n})$ of the play operators and the vector of the play operators

$$\boldsymbol{H}_{\boldsymbol{r}_H}[x,\boldsymbol{z}_{H0}](t)^T = (H_{r_{H0}}[x,z_{H00}](t) \quad H_{r_{H1}}[x,z_{H01}](t) \quad . \quad . \quad H_{r_{Hn}}[x,z_{H0n}](t)).$$

The hysteretic characteristic of the Prandtl-Ishlinskii hysteresis operator is completely defined by the characteristic of the so-called initial loading curve. This special branch will be traversed if the initial state of the Prandtl-Ishlinskii hysteresis operator is zero and it is driven with a monotonous increasing input signal. The initial loading curve can be fully characterized by and therefore equated with a threshold-dependent piecewise linear function

$$\varphi(r_H) = \sum_{j=0}^{i} w_{Hj}(r_H - r_{Hj}) \ ; \ \ r_{Hi} \le r_H < r_{Hi+1} \ \ ; \ \ i = 0\ .\ .\ n, \tag{6}$$

with $r_{Hn+1} = \infty$ and

$$\frac{\mathrm{d}}{\mathrm{d}r_H}\varphi(r_H) = \sum_{j=0}^{i} w_{Hj} \ \ ; \ \ r_{Hi} \le r_H < r_{Hi+1} \ \ ; \ \ i = 0\ .\ .\ n. \tag{7}$$

It is called the generator function of the Prandtl-Ishlinskii hysteresis operator [8], see Fig. 3 for a threshold-discrete Prandtl-Ishlinskii hysteresis operator with a model order of $n = 4$.

2.2 Compensation of Complex Hysteretic Nonlinearities

Under the consideration of the linear inequalitiy constraints

$$\boldsymbol{U}_H \cdot \boldsymbol{w}_H - \boldsymbol{u}_H \leq \boldsymbol{0} \tag{8}$$

for the weights with the matrix

$$\boldsymbol{U}_H = \begin{pmatrix} -1 & 0 & . & . & 0 \\ 0 & -1 & . & . & 0 \\ . & . & . & & . \\ . & . & & . & . \\ 0 & 0 & . & . & -1 \end{pmatrix}, \qquad \text{the vector} \qquad \boldsymbol{u}_H = \begin{pmatrix} -\varepsilon \\ 0 \\ . \\ . \\ 0 \end{pmatrix}$$

and a possibly infinite small number $\varepsilon > 0$ the generator function is strongly monotonous for $r_H \geq 0$ and therefore the inverse of the generator function $\varphi^{-1}(r_H)$ exists uniquely for $r_H \geq 0$. $\varphi^{-1}(r_H)$ is piecewise linear and strongly monotonous and can therefore also be regarded as a generator function

$$\varphi'(r'_H) = \sum_{j=0}^{i} w'_{Hj}(r'_H - r'_{Hj}) \;\; ; \;\; r'_{Hi} \leq r'_H < r'_{Hi+1} \;\; ; \;\; i = 0 \,.\,.\, n, \tag{9}$$

of a threshold-discrete Prandtl-Ishlinskii hysteresis operator with $r_{Hn+1}' = \infty$ and

$$\frac{\mathrm{d}}{\mathrm{d}r'_H}\varphi(r'_H) = \sum_{j=0}^{i} w'_{Hj} \;\; ; \;\; r'_{Hi} \leq r'_H < r'_{Hi+1} \;\; ; \;\; i = 0 \,.\,.\, n, \tag{10}$$

namely the inverse threshold-discrete Prandtl-Ishlinskii hysteresis operator

$$H^{-1}[y](t) := \boldsymbol{w}'^{T}_H \cdot \boldsymbol{H}_{r'_H}[y, \boldsymbol{z}'_{H0}](t) \tag{11}$$

with transformed initial states $\boldsymbol{z}_{H0}'$, threshold values $\boldsymbol{r}_H'$ and weights $\boldsymbol{w}_H'$. The transformation law $\boldsymbol{r}_H' = \boldsymbol{\Omega}_H(\boldsymbol{r}_H, \boldsymbol{w}_H)$ for the thresholds results from the relation $r'_{Hi} = \varphi(r_{Hi})$. From this follows

$$r'_{Hi} = \sum_{j=0}^{i} w_{Hj}(r_{Hi} - r_{Hj}) \quad ; \quad i = 0 \,.\,.\, n \tag{12}$$

for the threshold-discrete case, see Fig. 4. The transformation law $\boldsymbol{w}_H' = \boldsymbol{\Xi}_H(\boldsymbol{w}_H)$ for the weights results from the relation $\frac{\mathrm{d}}{\mathrm{d}r_H'}\varphi(r_{Hi}') = 1 \Big/ \frac{\mathrm{d}}{\mathrm{d}r_H}\varphi(r_{Hi})$, see Fig. 4. From this follows

$$w_{H0}' = \frac{1}{w_{H0}} \quad \text{and} \quad w_{Hi}' = -\frac{w_{Hi}}{(w_{H0} + \sum_{j=1}^{i} w_{Hj})(w_{H0} + \sum_{j=1}^{i-1} w_{Hj})} \quad ; \quad i = 1 \,..\, n. \tag{13}$$

The transformation law $\boldsymbol{z}_{H0}' = \boldsymbol{\Psi}_H(\boldsymbol{z}_{H0}, \boldsymbol{w}_H)$ for the initial states results from the relation $\frac{z_{H0i+1}' - z_{H0i}'}{r_{Hi+1}' - r_{Hi}'} = \frac{z_{H0i+1} - z_{H0i}}{r_{Hi+1} - r_{Hi}}$ between the initial states and the corresponding thresholds which is the threshold-discrete counterpart to the relation $\frac{\mathrm{d}}{\mathrm{d}r_H'} z(r_H') = \frac{\mathrm{d}}{\mathrm{d}r_H} z(r_H)$ for the threshold-continuous case discussed in [8]. From this follows

$$z_{H0i}' = \sum_{j=0}^{i} w_j z_{H0i} + \sum_{j=i+1}^{n} w_j z_{H0j} \quad ; \quad i = 0 \,..\, n \tag{14}$$

as the transformation law for the initial states.

2.3 Properties of the Prandtl-Ishlinskii Approach

The Prandtl-Ishlinskii hysteresis operator provided with the inequality constraints (8) for the weights has the following more or less obvious properties:

1. Because the Madelung's rules persist under linear superposition, they hold also for the threshold-discrete Prandtl-Ishlinskii hysteresis operator. Moreover, due to the $n > 1$ inner hysteretic state variables different branches can be traversed from a non turning point D which is in agreement with rule 4. This property agrees at least qualitatively with experimental observations for complex hysteretic nonlinearities.
2. The consideration of the inequality constraints (8) guarantees the invertibility of the threshold-discrete Prandtl-Ishlinskii hysteresis operator and leads to a convex generator function φ. Consequently, the increasing branches of the hysteresis operator are always convex and thus the hysteresis loops are always counterclockwise oriented. Moreover the

invertible Prandtl-Ishlinskii hysteresis operator described by the definition equation (5) and the inequality constraints (8) depends linear on the weights. For given thresholds the adaption of the model to the measured output-input data can be formulated as the minimization of the L_2^2-norm of the difference between the model output $H[x](t)$ and the output signal $y(t)$. This leads to a quadratic optimization problem for the weights with linear inequality constraints which has only one global solution. This guarantees a unique best L_2^2-norm approximation of the measured hysteretic characteristic in that space of the weights which leads to an invertible threshold-discrete Prandtl-Ishlinskii hysteresis operator with convex and couterclockwise loop orientation.

3. The inversion operation which is given by the transformation laws does not change the structure of the threshold-discrete Prandtl-Ishlinskii hysteresis operator. This leads to a direct formulation and thus to a very efficient implementation of the corresponding compensator which is profitable for real-time control applications.
4. The closed loops which will be traversed for input signals oscillating between maximum and minimum values have an odd symmetry to the center point of the corresponding loop. This so-called odd symmetry property is a property of the play operator and persists also under linear superposition.

The convexity property 2 which follows from the inequality constraints (8) and the odd symmetry property 3 which is an inherent model characteristic are the main drawbacks of this Prandtl-Ishlinskii modeling approach because they are often too restrictive for real complex hysteretic nonlinearities. An intuitive idea to overcome these restrictions is to combine in series the hysteresis operator and a continuous, non convex and non symmetrical memory-free nonlinearity. The main problem in this extension is to save the robustness of the design procedure for compensators of complex hysteretic nonlinearities presented before. The key is to find a modeling approach for complex memory-free nonlinearities which has the same pleasant properties relating to the identification and inversion operations as the Prandtl-Ishlinskii modeling approch for complex hysteretic nonlinearities.

3 Prandtl-Ishlinskii Modeling and Compensation of Memory-free Nonlinearities

In the mathematical literature memory-free nonlinearities are described by so-called superposition operators. The notation memory-free means that the present output signal value of the corresponding system depends only on the present input signal value. Therefore, superposition operators can be fully defined by functions. Modeling approaches for general continuous and perhaps differentiable memory-free nonlinearities which use the linear weighted superposition of elementary superposition operators defined by elementary functions are known for a long time, see for example power series expansion, Fourier series expansion etc.

3.1 Modeling of Memory-free Nonlinearities

In this paper another special modeling technique is used which is very close to the Prandtl-Ishlinskii modeling approach for complex hysteretic nonlinearities and coincides with it in a wide range. It bases on the weighted superposition of so-called one-sided dead-zone operators

$$y(t) = S_{r_S}[x](t) \tag{15}$$

which are defined by the relation

$$y(t) = S(x(t), r_S) \tag{16}$$

with the one-sided dead-zone function

$$S(x(t), r_S) = \begin{cases} \max\{x(t) - r_S, 0\} & ; \; r_S > 0 \\ x(t) & ; \; r_S = 0 \\ \min\{x(t) - r_S, 0\} & ; \; r_S < 0 \end{cases} \tag{17}$$

between the present values of the corresponding output and input signal. This elementary superposition operator is also fully characterized by a threshold parameter $r_S \in \Re$. Fig. 5

shows the rate-independent output-input trajectory of this elementary superposition operator for different threshold values.

Thus the complex superposition operator for the approximation of more general continuous memory-free nonlinearities is given by

$$S[x](t) := \boldsymbol{w}_S{}^T \cdot \boldsymbol{S}_{\boldsymbol{r}_S}[x](t) \tag{18}$$

with the vector of weights $\boldsymbol{w}_S{}^T = (w_{S-l} \,..\, w_{S-1}\; w_{S0}\; w_{S1} \,..\, w_{Sl})$, the vector of thresholds $\boldsymbol{r}_S{}^T = (r_{S-l} \,..\, r_{S-1}\; r_{S0}\; r_{S1} \,..\, r_{Sl})$ with $-\infty < r_{S-l} < .. < r_{S-1} < r_{S0} = 0 < r_{S1} < .. < r_{Sl} < +\infty$ and the vector of the one-sided dead-zone operators

$$\boldsymbol{S}_{\boldsymbol{r}_S}[x](t)^T = (S_{r_{S-l}}[x](t) \;\cdots\; S_{r_{S-1}}[x](t) \;\; S_{r_{S0}}[x](t) \;\; S_{r_{S1}}[x](t) \;\cdots\; S_{r_{Sl}}[x](t)).$$

Because of its high similarity to the threshold-discrete Prandtl-Ishlinskii hysteresis operator it is called the threshold-discrete Prandtl-Ishlinskii superposition operator. It should be mentioned that in contrast to the Prandtl-Ishlinskii hysteresis operator the Prandtl-Ishlinskii superposition operator encloses also elementary operators with negative thresholds. For this reason the Prandtl-Ishlinskii superposition operator is also able to approximate continuous memory-free nonlinearities which are not odd symmetrically to the origin.

An inherent requirement for every actuator or sensor which behaves in the quasistatic range like a continuous memory-free nonlinearity is the strong monotonicity of its output-input trajectory. This requirement can be considered by the linear inequality constraints for the weights

$$\boldsymbol{U}_S \cdot \boldsymbol{w}_S - \boldsymbol{u}_S \leq \boldsymbol{0} \tag{19}$$

with the matrix

$$\boldsymbol{U}_S = \begin{pmatrix} -1 & \cdots & -1 & -1 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & ⋰ & \vdots \\ 0 & \cdots & -1 & -1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & -1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & -1 & -1 & \cdots & 0 \\ \vdots & ⋰ & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & -1 & -1 & -1 & -1 \end{pmatrix} \quad \text{and the vector} \quad \boldsymbol{u}_S = \begin{pmatrix} -\varepsilon \\ \vdots \\ -\varepsilon \\ -\varepsilon \\ -\varepsilon \\ \vdots \\ -\varepsilon \end{pmatrix}$$

because they guarantee the strong monotonicity of the unique piecewise linear output-input trajectory of the threshold-discrete Prandtl-Ishlinskii superposition operator.

3.2 Compensation of Memory-free Nonlinearities

In exactly the same way as for the initial loading curve of threshold-discrete Prandtl-Ishlinskii hysteresis operator two threshold-dependent generator functions $\varphi_+(r_S)$ and $\varphi_-(r_S)$ can be defined for the unique positive and negative branch of the threshold-discrete Prandtl-Ishlinskii superposition operator. See Fig. 6 for a threshold-discrete Prandtl-Ishlinskii superposition operator with a model order of $l = 4$.

And with exactly the same arguments as before we get the following results: In the whole space of the weights which suffices the inequality constraints (19) the inverse of the threshold-discrete Prandtl-Ishlinskii superposition operator exists uniquely and is also a threshold-discrete Prandtl-Ishlinskii superposition operator with transformed thresholds and weights. Thus it is given by

$$S^{-1}[y](t) := \boldsymbol{w}_S'^{\,T} \cdot \boldsymbol{S}_{r_S'}[y](t) \quad . \tag{20}$$

Moreover due to the strong monotonicity of the threshold-discrete Prandtl-Ishlinskii superposition operator the inverse operator is also strongly monotonous and thus the transformed weights fulfil also the linear inequality constraints

$$\boldsymbol{U}_S \cdot \boldsymbol{w}_S' - \boldsymbol{u}_S \le \boldsymbol{0}. \tag{21}$$

The transformation laws $\boldsymbol{r}_S' = \boldsymbol{\Omega}_S(\boldsymbol{r}_S, \boldsymbol{w}_S)$ for the thresholds and $\boldsymbol{w}_S' = \boldsymbol{\Xi}_S(\boldsymbol{w}_S)$ for the weights correspond to the transformation laws of the threshold-discrete Prandtl-Ishlinskii hysteresis operator in a specific manner. For the positive branch follows

$$r_{Si}' = \sum_{j=0}^{i} w_{Sj}(r_{Si} - r_{Sj}) \quad ; \quad i = 0 \,.\,.\, l \tag{22}$$

and

$$w_{S0}' = \frac{1}{w_{S0}} \qquad \text{and} \qquad w_{Si}' = -\frac{w_{Si}}{(w_{S0} + \sum_{j=1}^{i} w_{Sj})(w_{S0} + \sum_{j=1}^{i-1} w_{Sj})} \quad ; \quad i = 1 \,.\,.\, l. \tag{23}$$

and for the negative branch follows

$$r_{Si}' = \sum_{j=i}^{0} w_{Sj}(r_{Si} - r_{Sj}) \quad ; \quad i = -l \,.\,.\, 0 \tag{24}$$

and

$$w_{S0}' = \frac{1}{w_{S0}} \qquad \text{and} \qquad w_{Si}' = -\frac{w_{Si}}{(w_{S0} + \sum_{j=i}^{-1} w_{Sj})(w_{S0} + \sum_{j=i+1}^{-1} w_{Sj})} \quad ; \quad i = -l \,.\,.\, \text{-}1. \tag{25}$$

The Prandtl-Ishlinskii superposition operator provided with the inequality constraints (19) for the weights has also the properties 2 and 3 of the Prandtl-Ishlinskii hysteresis operator. But in contrast to the Prandtl-Ishlinskii hysteresis operator the inequality constraints (19) permits also output-input trajectories which are not convex and not odd symmetrically.

4 A Modified Prandtl-Ishlinskii Modeling and Compensation Approach

As mentioned at the end of the second section an intuitive idea to overcome the modeling restriction due to the convexity and odd symmetry property of the threshold-discrete Prandtl-Ishlinskii hysteresis operator is to combine the threshold-discrete Prandtl-Ishlinskii hysteresis operator and the threshold-discrete Prandtl-Ishlinskii superposition operator in series.

4.1 Modeling and Compensation

This leads to the so-called modified threshold-discrete Prandtl-Ishlinskii hysteresis operator which is mathematically described by

$$\Gamma[x](t) := \boldsymbol{w}_S{}^T \cdot \boldsymbol{S}_{\boldsymbol{r}_S}[\boldsymbol{w}_H{}^T \cdot \boldsymbol{H}_{\boldsymbol{r}_H}[x, \boldsymbol{z}_{H0}]](t). \tag{26}$$

The inverse modified threshold-discrete Prandtl-Ishlinskii hysteresis operator can be easily obtained by the inversion of the threshold-discrete Prandtl-Ishlinskii hysteresis operator and the threshold-discrete Prandtl-Ishlinskii superposition operator and an exchange of their order. From this follows

$$\Gamma^{-1}[y](t) = \boldsymbol{w}'^T_H \cdot \boldsymbol{H}_{\boldsymbol{r}'_H}[\boldsymbol{w}'^T_S \cdot \boldsymbol{S}_{\boldsymbol{r}'_S}[y], \boldsymbol{z}'_{H0}](t). \tag{27}$$

4.2 Identification

The identification procedure which is used to adapt the model to the real hysteretic nonlinearity or to adapt the compensator to the inverse hysteretic nonlinearity is divided into three parts. In the first part the absolute maximum value of the measured input signal and the maximum and minimum value of the measured output signal are determined. With these values the thresholds $\boldsymbol{r}_H$ and the initial states $\boldsymbol{z}_{H0}$ of the Prandtl-Ishlinskii hysteresis operator and the thresholds $\boldsymbol{r}_S'$ of the inverse Prandtl-Ishlinskii superposition operator are determined by the formulas

$$r_{Hi} = \frac{i}{n+1} \max_{t_0 \le t \le t_e}\{|x(t)|\} = \frac{i}{n+1}\|x\|_\infty \quad ; \quad i = 0 \,..\, n, \tag{28}$$

$$r'_{S0} = 0, \tag{29}$$

$$r'_{Si} = \frac{(i - \frac{1}{2})}{l} \max_{t_0 \le t \le t_e}\{y(t)\} \quad ; \quad i = 1 \,..\, l, \tag{30}$$

$$r'_{Si} = \frac{(i + \frac{1}{2})}{l} \min_{t_0 \le t \le t_e}\{y(t)\} \quad ; \quad i = -l \,..\, -1 \tag{31}$$

and

$$z_{H0i} = 0 \quad ; \quad i = 0\,.\,.\,n\,. \tag{32}$$

In this case (32) assumes the start of the hysteretic state evolution from the so-called demagnetized state. The identification of the weights $\boldsymbol{w}_H$ and $\boldsymbol{w}_S'$ of the Prandtl-Ishlinskii hysteresis operator and the inverse Prandtl-Ishlinskii superposition operator which is the object of the second part can be formulated as an $L_2^{\,2}$-norm minimization of the so-called generalized error model

$$E[x,y](t) := \begin{pmatrix} \boldsymbol{w}_H^{\,T} & \boldsymbol{w}_S'^{\,T} \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{H}_{\boldsymbol{r}_H}[x, \boldsymbol{z}_{H0}]](t) \\ -\boldsymbol{S}_{\boldsymbol{r}_S'}[y]](t) \end{pmatrix} \tag{33}$$

which is linear dependent on the weights. This leads to the quadratic optimization problem

$$\min\{\begin{pmatrix} \boldsymbol{w}_H^{\,T} & \boldsymbol{w}_S'^{\,T} \end{pmatrix} \int_{t_0}^{t_e} \begin{pmatrix} \boldsymbol{H}_{\boldsymbol{r}_H}[x, \boldsymbol{z}_{H0}]](t) \\ -\boldsymbol{S}_{\boldsymbol{r}_S'}[y]](t) \end{pmatrix} \begin{pmatrix} \boldsymbol{H}_{\boldsymbol{r}_H}[x, \boldsymbol{z}_{H0}]](t)^T & -\boldsymbol{S}_{\boldsymbol{r}_S'}[y]](t)^T \end{pmatrix} \mathrm{d}t \begin{pmatrix} \boldsymbol{w}_H \\ \boldsymbol{w}_S' \end{pmatrix}\} \tag{34}$$

with the linear inequality constraints

$$\begin{pmatrix} \boldsymbol{U}_H & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{U}_S \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{w}_H \\ \boldsymbol{w}_S' \end{pmatrix} - \begin{pmatrix} \boldsymbol{u}_H \\ \boldsymbol{u}_S \end{pmatrix} \leq \begin{pmatrix} \boldsymbol{o} \\ \boldsymbol{o} \end{pmatrix}. \tag{35}$$

This minimization problem is overdetermined with one degree of freedom because the elementary operators $H_{r_H}\big|_{r_H=0}$ and $S_{r_S'}\big|_{r_S'=0}$ are both equal to the identity operator I. The additional linear equality constraint

$$\begin{pmatrix} (\|x\|_\infty \cdot \boldsymbol{i} - \boldsymbol{r}_H)^T & \boldsymbol{o}^T \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{w}_H \\ \boldsymbol{w}_S' \end{pmatrix} - \|x\|_\infty = 0 \tag{36}$$

with the unity vector $\boldsymbol{i}^T = (1\ 1\ .\ .\ 1)$ deletes this degree of freedom and ensures the unique solvability of the quadratic minimization problem. This linear equality constraint leads to an amplitude range invariance property of the corresponding Prandtl-Ishlinskii hysteresis operator. Thus the mapping of the input amplitude range to the output amplitude range is only determined by the corresponding Prandtl-Ishlinskii superposition operator.

For the identification of the modified threshold-discrete Prandtl-Ishlinskii hysteresis operator the thresholds $\boldsymbol{r}_S$ and the weights $\boldsymbol{w}_S$ of the Prandtl-Ishlinskii superposition operator are calculated in the third part from the thresholds $\boldsymbol{r}_S'$ and the weights $\boldsymbol{w}_S'$ of the inverse Prandtl-Ishlinskii superposition operator by the given corresponding weight- and threshold transformation laws

$$\boldsymbol{r}_S = \boldsymbol{\Omega}(\boldsymbol{r}_S', \boldsymbol{w}_S')$$

and

$$\boldsymbol{w}_S = \boldsymbol{\Xi}(\boldsymbol{w}_S')$$

which are decribed by (22) - (25) in detail. In contrast to the identification of the modified threshold-discrete Prandtl-Ishlinskii hysteresis operator the identification of the inverse modified threshold-discrete Prandtl-Ishlinskii hysteresis operator requires the thresholds $\boldsymbol{r}_H'$, the weights $\boldsymbol{w}_H'$ and the initial states $\boldsymbol{z}_{H0}'$ of the inverse Prandtl-Ishlinskii hysteresis operator. They are calculated from the thresholds $\boldsymbol{r}_H$, the weights $\boldsymbol{w}_H$ and the initial states $\boldsymbol{z}_{H0}$ of the Prandtl-Ishlinskii hysteresis operator by the given corresponding weight-, threshold- and initial state transformation laws

$$\boldsymbol{r}_H' = \boldsymbol{\Omega}_H(\boldsymbol{r}_H, \boldsymbol{w}_H),$$

$$\boldsymbol{w}_H' = \boldsymbol{\Xi}_H(\boldsymbol{w}_H)$$

and

$$\boldsymbol{z}_{H0}' = \boldsymbol{\Psi}_H(\boldsymbol{z}_{H0}, \boldsymbol{w}_H)$$

which are decribed by (12) - (14) in detail. The inequality constraints (35) guarantee in the same way as the inequality constraints (8) for the Prandtl-Ishlinskii modeling approach the best L_2^2-norm minimization in that space of the weights which leads to invertible threshold-discrete Prandtl-Ishlinskii hysteresis and superposition operators. Therefore the invertibility of the modified threshold-discrete Prandtl-Ishlinskii hysteresis operator is always guaranteed during the optimization and thus the design process for the model or the compensator is consistent and robust against unknown measurement errors of input-output data, unknown model errors and unknown model orders. Thus the modified Prandtl-Ishlinskii modeling approach leads to an extension of the Prandtl-Ishlinskii modeling approach which permits the consistent modeling, identification and compensation of invertible complex hysteretic nonlinearities which have non convex increasing branches and non odd symmetrical loops.

The implementation of the described identification scheme uses a numerical solver for quadratic optimization problems with linear inequality and equality constraints. Such problems are well known as quadratic programs and are standard problems in the mathematical optimization theory. They are discussed with neccessary detail in the optimization literature [11]. Therefore the presentation of a explicit scheme for the solution of quadratic programs is beyond the scope of this paper.

5 Application to a Magnetostrictive Actuator

In this section the performance of the presented compensator design method for complex hysteretic nonlinearities will now be demonstrated by means of the displacement-current relation of a magnetostrictive transducer. Fig. 7 shows the typical major and minor butterfly loops of a magnetostrictive transducer if it is driven with current amplitudes of ± 2 A and a typical pre-force of 500 N.

To get a strongly monotonous relation between the displacement and current the magnetostrictive transducer is normally used with an additional bias current. The bias current which amounts to 1 A in this example determines together with the pre-force of 500 N the operating point of the magnetostrictive actuator. The electrical operating range is characterized by the grey area in Fig. 7 and amounts to 1 A ± 0,8 A. Thus we have a safety range of about 20 % to current ranges with a non monotonous displacement-current relation. Fig. 8 shows the strongly monotonous hysteretic displacement-current relation of the magnetostrictive actuator in the operating range. It is mainly characterized by non convex increasing branches and asymmetrical hysteretic loops with a counterclockwise orientation. Therefore the modeling, identification and compensation of this real complex hysteretic nonlinearity cannot be realized with the conventional Prandtl-Ishlinskii approach but requires the extensions introduced by the modified Prandtl-Ishlinskii approach.

Fig. 9 shows the looping and branching behaviour of the modified threshold-discrete Prandtl-Ishlinskii hysteresis operator with different model orders n and l as a result of the identification procedure. The model order n = 0 and l = 0 leads to a linear rate-independent operator model and thus the identification procedure determines the best linear $L_2{}^2$-norm approximation of the real hysteretic nonlinearity. The nonlinearity error defined by

$$\frac{\max\limits_{t_0 \le t \le t_e}\{|\Gamma[I](t) - s(t)|\}}{\max\limits_{t_0 \le t \le t_e}\{|\Gamma[I](t)|\}} \tag{37}$$

amounts in this case up to 48,72 %. Increasing the model order to n = 4 and l = 2 leads to a much better piecewise linear approximation of the major loop, but due to the low model order no branching occurs within the hysteretic region. The nonlinearity error is in this case reduced to 6,33 %. Increasing of the model order further to n = 8 and l = 4 the modified threshold-discrete Prandtl-Ishlinskii hysteresis operator is also able to approximate the minor loops with a higher accuracy. But also in this case the approximation of the real hysteretic nonlinearity with piecewise linear branches is visible. The nonlinearity error is in this case further reduced to 3,38 %. Finally a modified threshold-discrete Prandtl-Ishlinskii operator with a model order of n = 14 and l = 7 leads also to a sufficiently smooth approximation of the minor loops. The nonlinearity error amounts in this case to 2,61 % which is nearly twenty times smaller than for the best linear $L_2{}^2$-norm approximation. Due to small unknown measurement and model errors a further increasing of the model order n and l don’t reduce significantly the nonlinearity error. A general procedure for the user to obtain the right model complexity for a given complex hysteretic nonlinearity works with the strategy decribed before. Starting with a low model order, for example $n = 0$ and $l = 0$, the model order is increased sucessively until the defined nonlinearity error (37) saturates in dependence on the model order.

For the compensation of the real hysteretic nonlinearity a feedforward controller is used which is based on the inverse modified threshold-discrete Prandtl-Ishlinskii hysteresis operator, see Fig. 10. $s_c(t)$ is the given displacement signal value.

The inverse modified threshold-discrete Prandtl-Ishlinskii hysteresis operator is obtained from the modified threshold-discrete Prandtl-Ishlinskii hysteresis operator with the model order of $n = 14$ and $l = 7$ using the transformation laws for the thresholds, weights and initial states.

It is realized by a digital signal processor with a sampling rate of up to 10 kHz and a displacement controlled current source. The looping and branching characteristic of the inverse modified threshold-discrete Prandtl-Ishlinskii hysteresis operator is shown in Fig. 11.

As a final result Fig. 12 shows the compensated characteristic of the overall system given by the serial combination of the inverse feedforward controller and the magnetostrictive actuator. The input signal range for the compensator corresponds to the output signal range of the actuator. Tn this example the control error defined by

$$\frac{\max\limits_{t_0 \le t \le t_e}\{|s_c(t) - s(t)|\}}{\max\limits_{t_0 \le t \le t_e}\{|s_c(t)|\}} \tag{38}$$

will be strongly reduced to about 3 % due to the inverse feedforward control strategy.

6 Conclusions

The main contribution of this paper is to extend the Prandtl-Ishlinskii approach for the modeling, identification and compensation of complex hysteretic nonlinearities with convex branches and symmetrical hysteresis loops to a so-called modified Prandtl-Ishlinskii approach for the modeling, identification and compensation of complex hysteretic nonlinearities with non convex branches and asymmetrical hysteresis loops. For this purpose a modified Prandtl-Ishlinskii hysteresis operator is defined by the serial combination of a conventional Prandtl-Ishlinskii hysteresis operator and a memory-free nonlinearity with an asymmetrical graph. Based on this modeling method a robust compensator design procedure for invertible complex hysteretic nonlinearities is developed. Finally, the compensator design method is used to generate an inverse feedforward controller for a magnetostrictive actuator. In comparison to the conventional controlled magnetostrictive actuator the nonlinearity error of the inverse controlled magnetostrictive actuator is lowered from about 50 % to about 3 %.

In future works the modified Prandtl-Ishlinskii Modeling, Identification and Compensation approach for complex hysteretic nonlinearities will be extended to rate-dependent creep processes which play an important role especially in wideband applications of piezoelectric actuators.

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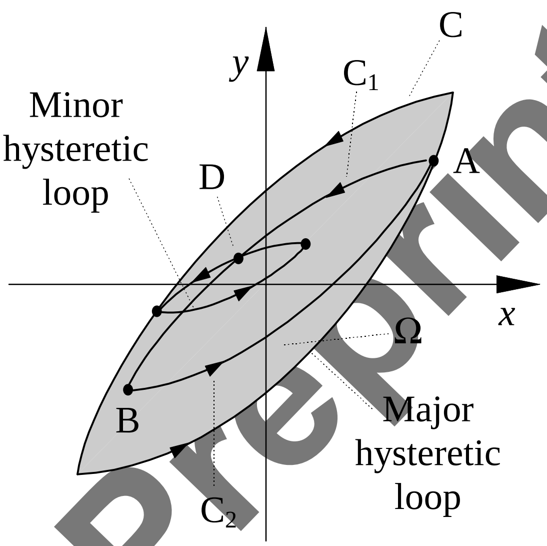


Fig. 1

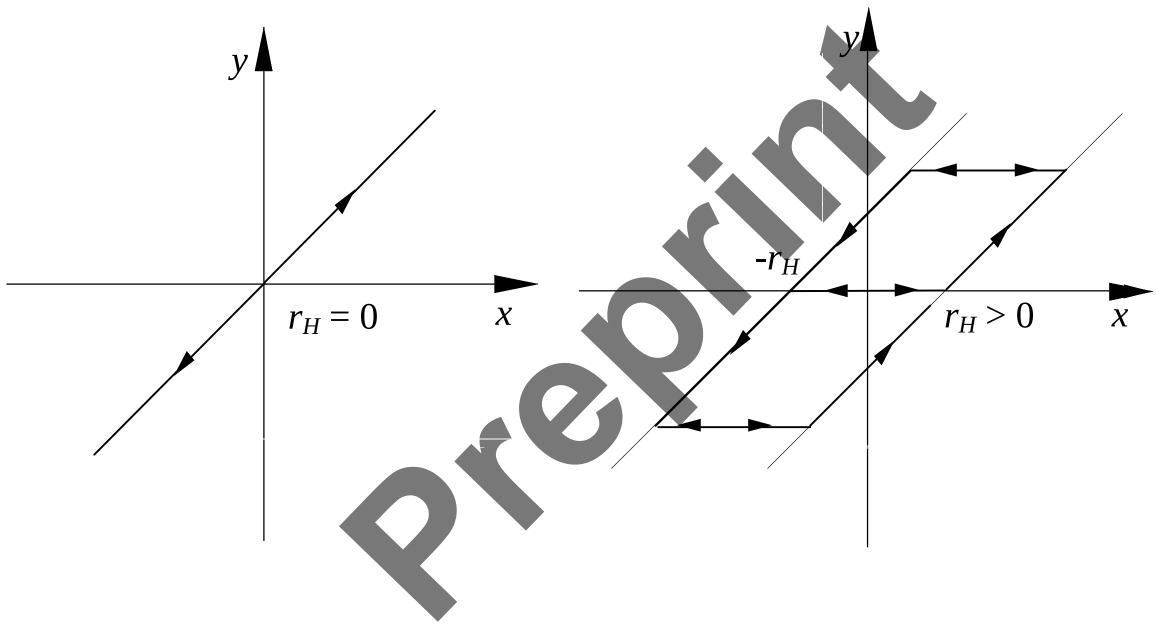


Fig. 2

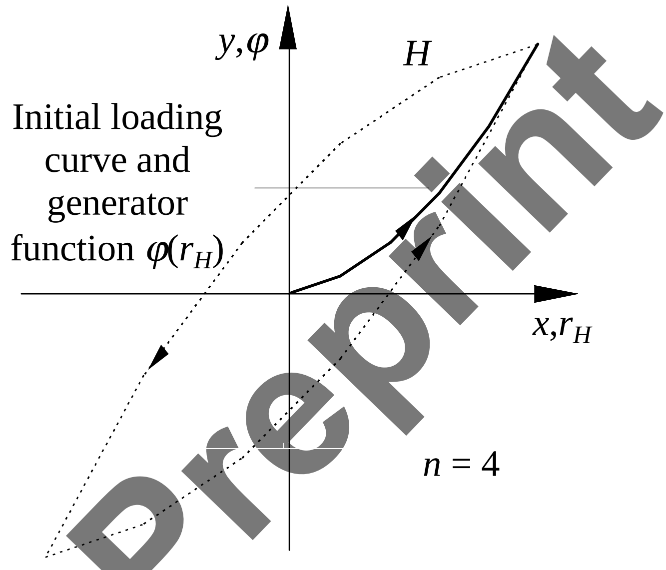
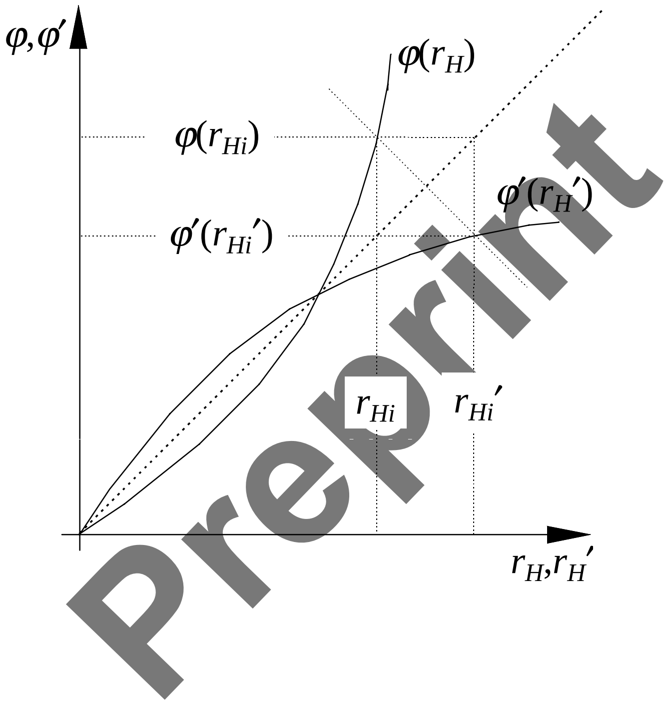
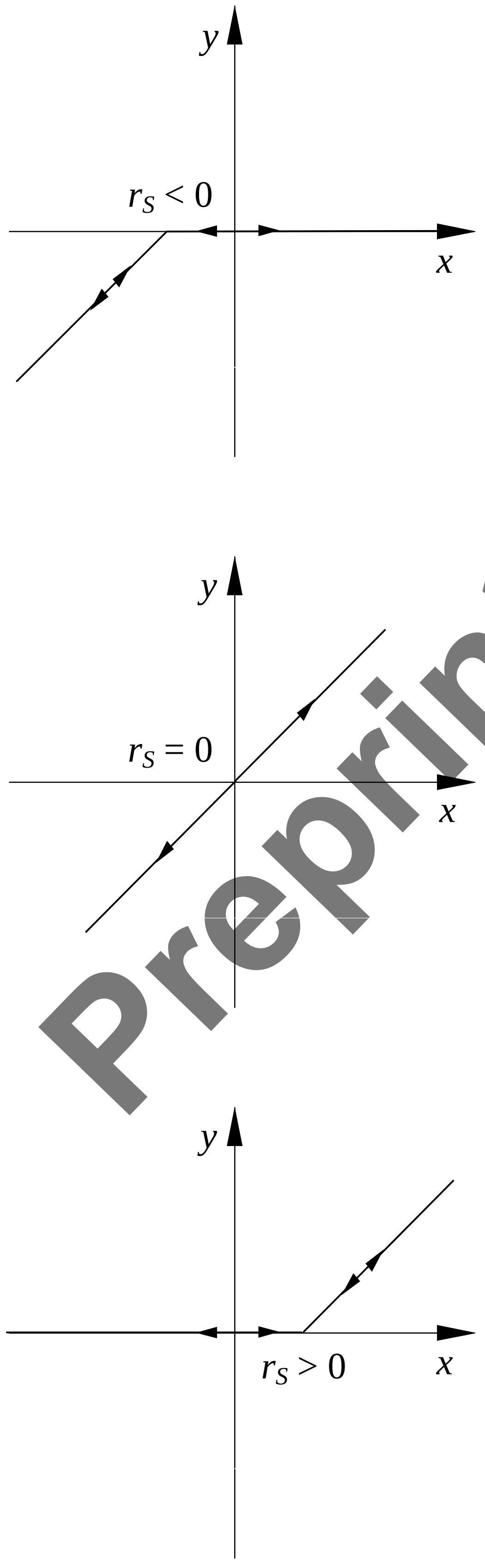

y,φ
H
Initial loading
curve and
generator
function φ(r_H)
x,r_H
n = 4

Fig. 3



φ,φ'
$\varphi(r_H)$
$\varphi(r_{Hi})$
$\varphi'(r_H')$
$\varphi'(r_{Hi}')$
r_{Hi}
r_{Hi}'
r_H,r_H'

Fig. 4



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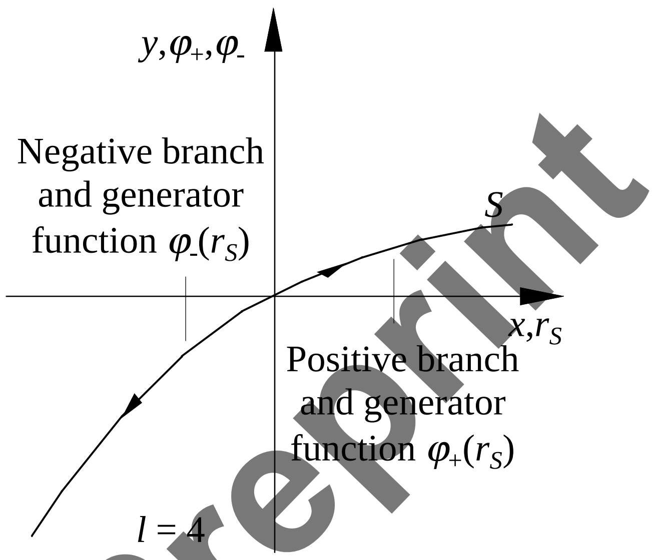


y,φ+,φ-
Negative branch
and generator
function φ-(rS)
S
x,rS
Positive branch
and generator
function φ+(rS)
l = 4

Fig. 6

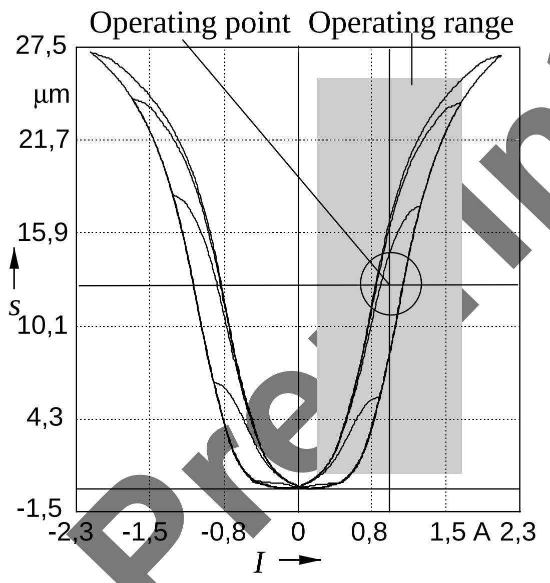

Operating point
Operating range
27,5
µm
21,7
15,9
s
10,1
4,3
-1,5
-2,3
-1,5
-0,8
0
0,8
1,5 A
2,3
I

Fig. 7

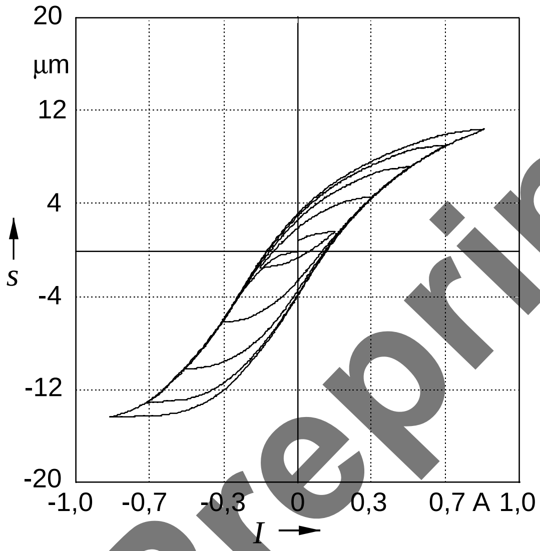


Fig. 8

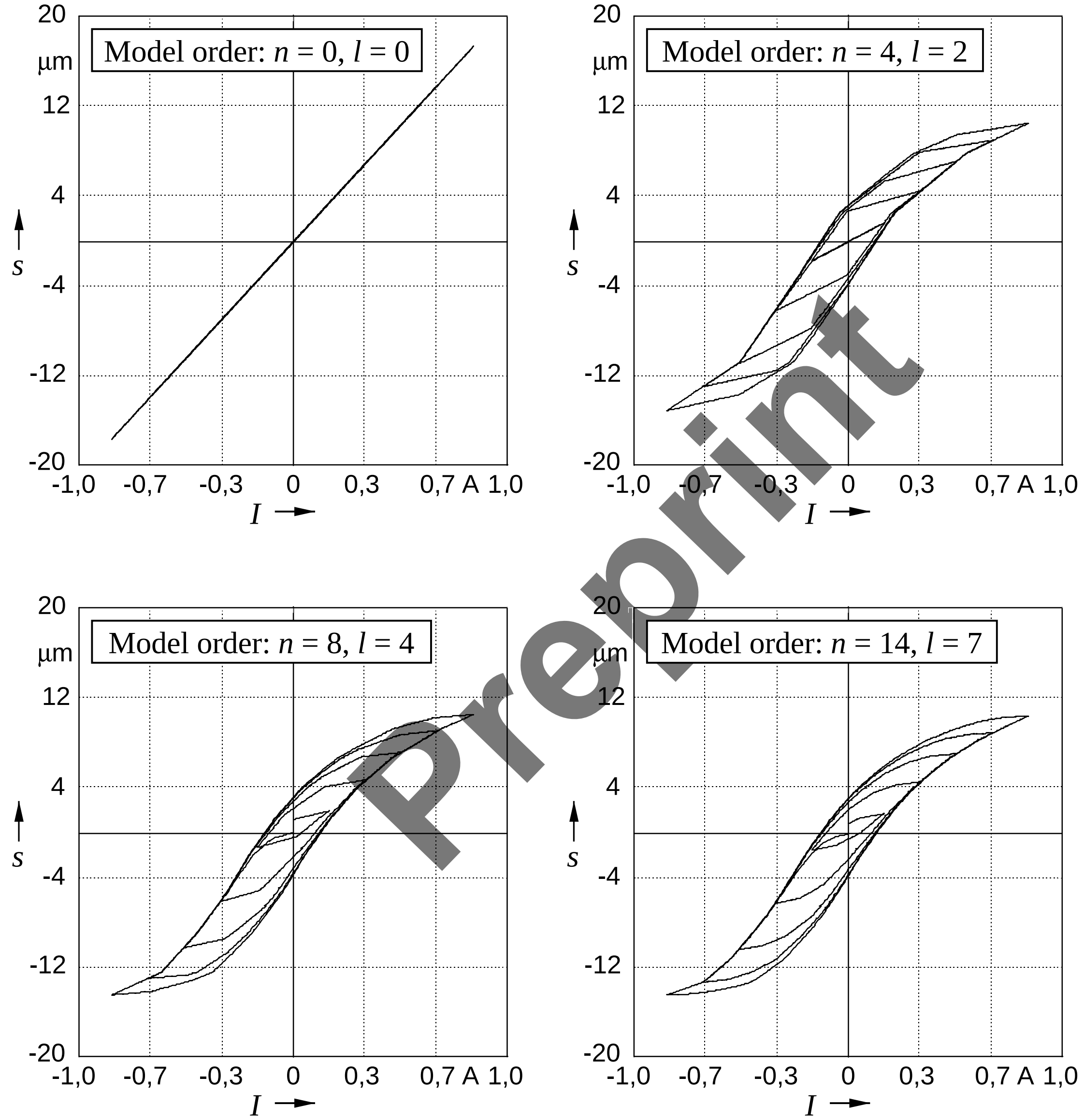
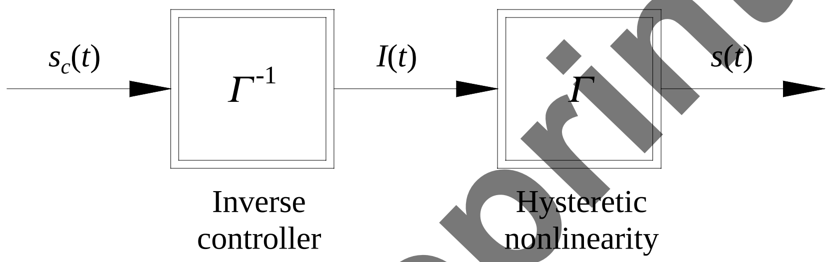

Model order: n = 0, l = 0
Model order: n = 4, l = 2
Model order: n = 8, l = 4
Model order: n = 14, l = 7
20
µm
12
4
-4
-12
-20
s
-1,0
-0,7
-0,3
0
0,3
0,7 A
1,0
I

Fig. 9



$s_c(t)$
Γ^{-1}
$I(t)$
Γ
$s(t)$
Inverse controller
Hysteretic nonlinearity

Fig. 10

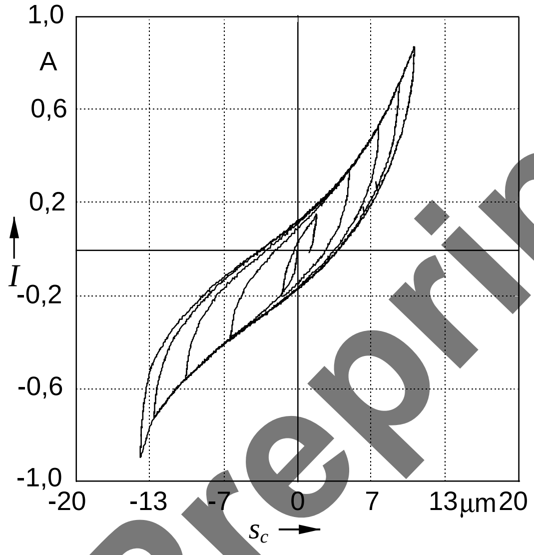


Fig. 11

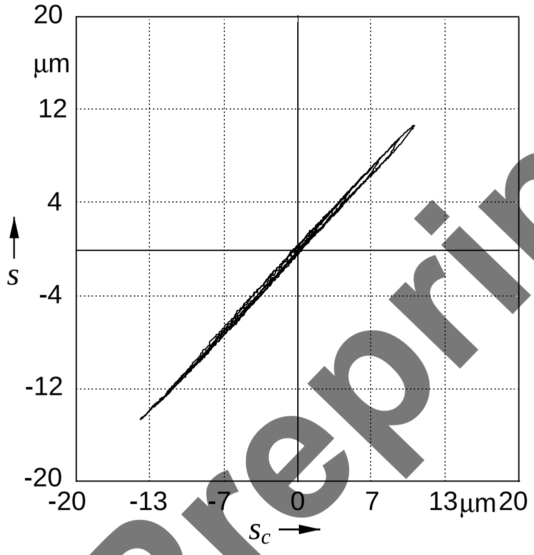


Fig. 12

Fig. 1: Complex hysteretic nonlinearity

Fig. 2: Rate-independent characteristic of the play operator

Fig. 3: Initial loading curve and generator function $\varphi(r_H)$

Fig. 4: Generator functions $\varphi(r_H)$ and $\varphi'(r_H')$

Fig. 5: Rate-independent characteristic of the one-sided dead-zone operator

Fig. 6: Positive and negative branch and generator functions

Fig. 7: Butterfly-loops of a magnetostrictive transducer

Fig. 8: Measured hysteretic displacement-current relation in the operating range

Fig. 9: Modeled hysteretic displacement-current relation in the operating range

Fig. 10: Compensation of the hysteretic nonlinearitiy by an inverse feedforward controller

Fig. 11: Inverted hysteretic displacement-current relation in the operating range

Fig. 12: Compensated hysteretic displacement-current relation in the operating range