

## Determining the electromechanical characteristics of piezoelectric stack actuators during life-cycle testing

M. Klein, K. Kuhnen, M. Rissing, and H. Janocha  
 Laboratory for Process Automation  
 Saarland University, Saarbrücken (Ger)

### 1 Abstract

Many applications of piezo actuator systems require dynamic operation with large forces and maximum displacement (large-signal operation). The service life of these transducers must be comparable to that of classical drives. However, small-signal parameters are insufficient for drawing conclusions about the transducer integrity. For nearly any application, examinations of the specific conditions have to be conducted.

The presented method allows simultaneous measuring of the leakage resistance, the capacitance, the stiffness and the piezoelectric constants of a piezoelectric actuator in large-signal operation. The method is used in a life-cycle test bench. The electrical and mechanical driving signals can be defined separately as long as they are linearly independent periodic signals. By monitoring the periodic changes in the parameters mentioned above the "history" of a piezoelectric actuator can be documented and retraced to the point of its failure.

### 2 Introduction

Piezoelectric actuators, as high-precision and extremely fast controlling elements, are used in a growing number of fields and have even found their way into the mass market. Producers and users alike are interested in knowing about the service life and reliability of piezoelectric actuators as their use is only justified if their correct functioning can be guaranteed. The operating behaviour of piezoelectric actuators can be influenced by the environment, e.g. by high temperatures, high humidity or pollution. A decrease in leakage resistance, for instance, is caused by electrically conductive channels spreading out within the ceramic due to material breakdown or by electrically conductive paths emerging at the surface of the ceramic due to water vapour absorption [1]. The leakage resistance can decrease so far as to overload the power electronics and to disrupt the required input-output behaviour [2].

Typically, the leakage resistance is measured statically by means of applying a DC voltage [3]. Especially for dynamic long-term tests in which the actuators are driven electrically and/or mechanically with periodic signals to measure the capacitance or the strain, for instance, this static measuring approach would require modified measuring equipment and temporary interruption of the dynamic cycling.

In comparison to a formerly described measuring method [5] which was developed for the diagnosis of only the electrical characteristics, the method introduced here allows the simultaneous measuring of the electrical, mechanical and electromechanical parameters in large-signal long-term tests. The test equipment need not be altered. For deriving the algorithm the piezoelectric stack transducer is modelled as an electromechanical quadripole. The linear quadripole model parameters leakage resistance, capacitance, stiffness and piezoelectric charge constants are computed using the least-squares method.

### 3 Test device and measuring methods

The automatic test and measurement device consists of a PC with integrated signal processing electronics, a power amplifier for driving piezoelectric actuators, a power source to generate mechanical loads, a so-called Sawyer-Tower circuit [4] to detect the electric charge and voltage of the device under test (DUT) and sensors to register the mechanical parameters force and displacement (Fig. 1). The control program defines the input voltages  $V_{in}$  and  $V_{Fin}$  for the voltage source and the power source which generate the voltage  $V$  and the force  $F_P$ , respectively. The modified Sawyer-Tower circuit (Fig. 2) includes the DUT. The amplifier output voltage  $V$ , the measuring voltage  $V_M$ , the actuator displacement  $S_P$  and the effective force  $F_P$  are simultaneously measured and transferred into the PC.

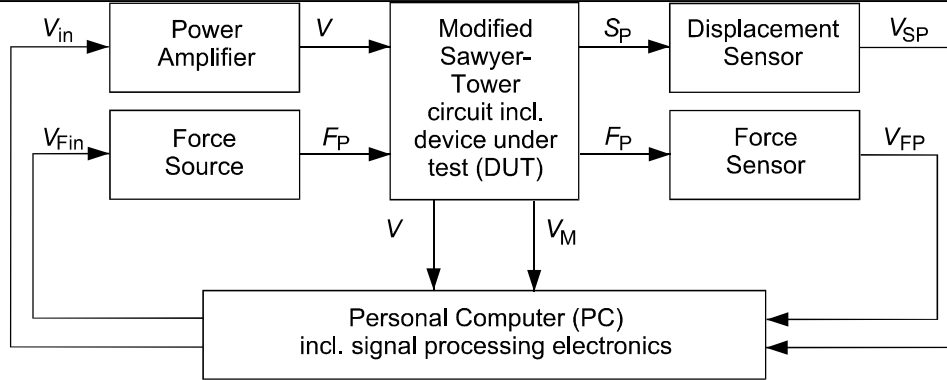


Figure 1: Automatic measuring and test device

The DUT which is contained in the modified Sawyer-Tower circuit (Fig. 2) is modelled as a quadripole (grey-shadowed). It is connected in series with the reference impedance ( $C_M \parallel R_M$ ). The electrical properties are described by the capacitance  $C_P$  and the leakage resistance  $R_P$ , the mechanical properties by the stiffness  $c_P$  and the piezoelectric ones by the actuator constant  $d_A$  and the sensor constant  $d_S$ . The two last-mentioned parameters  $d_A$  and  $d_S$  are actuator specific parameters for the large-signal operating mode. They have to be distinguished from the piezo modulus  $d_{33}$  which is a material specific parameter defined for the small-signal operating mode.

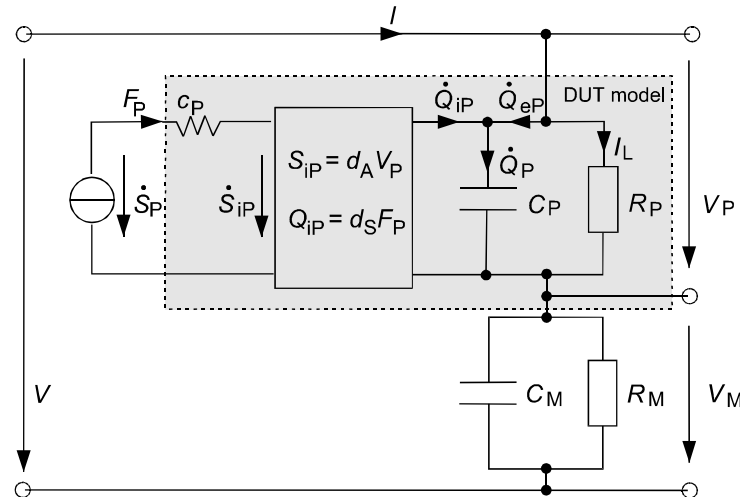


Figure 2: Modified Sawyer-Tower-measurement circuit

The output current  $I$  supplied by the voltage source is divided into the leakage current  $I_L$  which results from the leakage resistance  $R_P$ , and  $dQ_{eP}/dt$  which charges and discharges the actuator capacitance  $C_P$ . The force  $F_P$  which loads the actuator surface generates the electric charge  $Q_{iP}$ , as a result of the direct piezoelectric effect. Thus the overall polarisation charge  $Q_P$  stored in the piezoelectric actuator amounts to  $Q_P = Q_{eP} + Q_{iP}$ . The voltage  $V_P$  applied to the piezoelectric actuator causes the displacement  $S_{iP}$  due to the indirect piezoelectric effect. The effective displacement  $S_P$  of the actuator results from the superposition of the actuator operation and the external mechanical load.

### 3.1 Function of the modified Sawyer-Tower circuit

With regard to the intended application, the system properties of the modified Sawyer-Tower circuit are described via the parameters defined in Fig. 2 as follows. Within the frequency domain, the measured voltage can be described as

$$\underline{V}_M(j\omega) = \frac{R_M}{R_M + R_P} \underline{A}(j\omega) \underline{V}(j\omega) + \frac{R_M R_P C_P}{R_M + R_P} \underline{B}(j\omega) \underline{V}(j\omega) + \frac{R_M R_P d_S}{R_M + R_P} \underline{B}(j\omega) \underline{F}_P(j\omega) \quad (1)$$

and the filtered displacement by

$$\underline{S}_M(j\omega) = \underline{C}(j\omega) \underline{S}_P(j\omega) = \underline{C}(j\omega) d_A (\underline{V}(j\omega) - \underline{V}_M(j\omega)) + \frac{1}{C_P} \underline{C}(j\omega) \underline{F}_P(j\omega) \quad (2)$$

with

$$\underline{A}(j\omega) = \frac{1}{1 + j\omega \frac{R_P R_M (C_M + C_P)}{R_M + R_P}}, \quad (3)$$

$$\underline{B}(j\omega) = \frac{j\omega}{1 + j\omega \frac{R_P R_M (C_M + C_P)}{R_M + R_P}} \quad (4)$$

and

$$\underline{C}(j\omega) = \frac{j\omega}{1 + j\omega R_M C_M}. \quad (5)$$

Assuming the given voltage  $V(t)$  and the given force  $F_P(t)$  are periodic with the cycle durations  $T_{V0}$  and  $T_{F0}$ , respectively, for the voltage follows

$$V(t) = V(t + T_{V0}) \quad (6)$$

and for the force

$$F_P(t) = F_P(t + T_{F0}). \quad (7)$$

Furthermore, it is assumed that the given voltage  $V(t)$  and the given force  $F_P(t)$  can be split into a constant component and an alternating component resulting in

$$V(t) = V_0 + V_-(t) \quad (8)$$

and

$$F_P(t) = F_{P0} + F_{P-}(t). \quad (9)$$

Consequently, the amplitude spectrum  $|\underline{V}(j\omega)|$  of the given voltage consists of discrete spectral lines at  $\omega_{V0} = 0, \omega_{V1} = 2\pi/T_{V0}, \dots, \omega_{Vn} = 2\pi n/T_{V0}, \dots$ . The same holds for the amplitude spectrum  $|\underline{F}_P(j\omega)|$  of the given force with the discrete spectral lines at the angular frequencies  $\omega_{F0} = 0, \omega_{F1} = 2\pi/T_{F0}, \dots, \omega_{Fn} = 2\pi n/T_{F0}, \dots$ . Since all other signals within the circuit are formed as a reaction to the given voltage  $V(t)$  and the given force  $F_P(t)$ , their amplitude spectra are composed of the linear superposition of the voltage and the force spectra. The discrete spectral lines are at  $\omega_0 = 0, \omega_{V1} = 2\pi/T_{V0}, \omega_{F1} = 2\pi/T_{F0}, \dots, \omega_{Vn} = 2\pi n/T_{V0}, \omega_{Fn} = 2\pi n/T_{F0}, \dots$  (Figure 3). If the reference resistance  $R_M$  is significantly smaller than the leakage resistance of the piezo actuator and if the measuring capacitance  $C_M$  is significantly higher than the capacitance  $C_P$  of the piezo actuator, i.e.  $R_M \ll R_P$  and  $C_M \gg C_P$ , then

$(R_M + R_P)/(R_M R_P (C_M + C_P)) \approx 1/(R_M C_M)$  and the 3dB cut-off frequencies of the filters  $\underline{A}(j\omega)$  and  $\underline{B}(j\omega)$  correspond approximately with the 3dB cut-off frequency  $\omega_G = 1/(R_M C_M)$  of the filter  $\underline{C}(j\omega)$ .

If the fundamental frequencies  $\omega_{V1} = 2\pi/T_{V0}$  of the given voltage  $V(t)$  and  $\omega_{F1} = 2\pi/T_{F0}$  of the given force  $F_P(t)$  are clearly higher than  $\omega_G$ , i.e.  $\omega_{V1}, \omega_{F1} \gg \omega_G$ , then the filter  $\underline{A}(j\omega)$  shown in Figure 3 acts like a low-pass filter which filters the alternating components out of the amplitude spectrum  $|\underline{V}_M(j\omega)|$  of the measuring voltage. Whereas filters  $\underline{B}(j\omega)$  and  $\underline{C}(j\omega)$  shown in Figure 3 below behave like high-pass filters which filter the constant component out of the amplitude spectrum  $|\underline{V}_M(j\omega)|$  of the measuring voltage and out of the amplitude spectrum  $|\underline{S}_M(j\omega)|$  of the filtered displacement. Thus the constant and alternating components of the signals can be analysed separately.

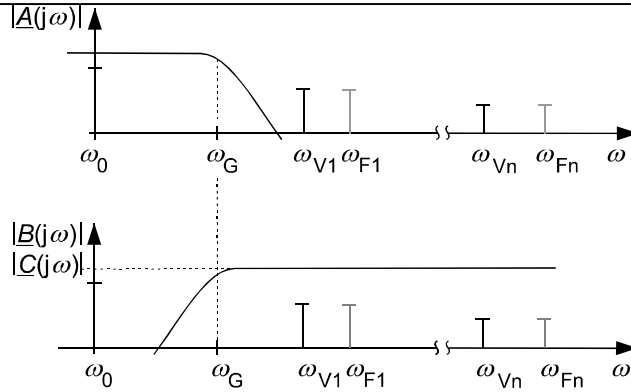


Figure 3: Amplitude characteristics  $|A(j\omega)|$  and  $|B(j\omega)| \approx |C(j\omega)|$  with  $C_M \gg C_P$  and  $R_M \ll R_P$

### 3.2 Measuring method

Considering the predefined conditions and based on (1) - (3), the constant component of the measuring voltage can be derived to

$$V_{M0} = \frac{R_M}{R_M + R_P} V_0. \quad (10)$$

Additionally, the alternating components of the measuring voltage and the filtered displacement result to

$$V_{M-}(t) = \frac{C_P}{C_M + C_P} V_-(t) + \frac{d_S}{C_M + C_P} F_{P-}(t) \quad (11)$$

and

$$S_{M-}(t) = \frac{d_A}{R_M C_M} (V_-(t) - V_{M-}(t)) + \frac{1}{c_P R_M C_M} F_{P-}(t). \quad (12)$$

When the sample values of the signals  $V(t)$ ,  $F_P(t)$ ,  $V_M(t)$  and  $S_M(t)$  are combined into  $n$ -dimensional signal vectors  $\mathbf{V}$ ,  $\mathbf{F}_P$ ,  $\mathbf{V}_M$  and  $\mathbf{S}_M$ , and the measuring time  $T_M$  is equivalent to an integer multiple of the cycle durations  $T_{V0}$  and  $T_{F0}$ , the constant components of the signals  $V_0$ ,  $F_{P0}$ ,  $V_{M0}$  and  $S_{M0}$  can be calculated by simple averaging. The alternating components  $\mathbf{V}_-$ ,  $\mathbf{F}_{P-}$ ,  $\mathbf{V}_{M-}$  and  $\mathbf{S}_{M-}$  can be computed by subtracting the respective constant components of the signal vectors  $\mathbf{V}$ ,  $\mathbf{F}_P$ ,  $\mathbf{V}_M$  and  $\mathbf{S}_M$ . Together with equations (11) and (12) that leads to the over-determined system of equations

$$(\mathbf{V}_- \quad \mathbf{F}_{P-}) \begin{pmatrix} \frac{C_P}{C_M + C_P} \\ \frac{d_S}{C_M + C_P} \end{pmatrix} = \mathbf{V}_{M-} \quad (13)$$

and

$$((\mathbf{V}_- - \mathbf{V}_{M-}) \quad \mathbf{F}_{P-}) \begin{pmatrix} \frac{d_A}{R_M C_M} \\ 1 \\ \frac{1}{c_P R_M C_M} \end{pmatrix} = \mathbf{S}_{M-}. \quad (14)$$

This system of equations can be handled using the methods of regression analysis (least-squares method). The solutions of this system of equations are the so-called normal equations

$$\begin{pmatrix} \frac{C_P}{C_M + C_P} \\ \frac{d_S}{C_M + C_P} \end{pmatrix} = \left( \begin{pmatrix} \mathbf{V}_{\sim}^T \\ \mathbf{F}_{P\sim}^T \end{pmatrix} (\mathbf{V}_{\sim} \quad \mathbf{F}_{P\sim}) \right)^{-1} \begin{pmatrix} \mathbf{V}_{\sim}^T \\ \mathbf{F}_{P\sim}^T \end{pmatrix} \mathbf{V}_{M\sim} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad (15)$$

and

$$\begin{pmatrix} \frac{d_A}{R_M C_M} \\ \frac{1}{c_P R_M C_M} \end{pmatrix} = \left( \begin{pmatrix} (\mathbf{V}_{\sim} - \mathbf{V}_{M\sim})^T \\ \mathbf{F}_{P\sim}^T \end{pmatrix} ((\mathbf{V}_{\sim} - \mathbf{V}_{M\sim}) \quad \mathbf{F}_{P\sim}) \right)^{-1} \begin{pmatrix} (\mathbf{V}_{\sim} - \mathbf{V}_{M\sim})^T \\ \mathbf{F}_{P\sim}^T \end{pmatrix} \mathbf{S}_{M\sim} = \begin{pmatrix} a_3 \\ a_4 \end{pmatrix}. \quad (16)$$

These normal equations together with equation (10) enable the determination of the searched quadripole parameters depending on the given measuring circuit parameters  $R_M$  and  $C_M$

$$R_P = R_M \frac{V_0 - V_{M0}}{V_{M0}}, \quad (17)$$

$$C_P = C_M \frac{a_1}{1 - a_1}, \quad (18)$$

$$d_S = C_M \frac{a_2}{1 - a_1}, \quad (19)$$

$$d_A = R_M C_M a_3, \quad (20)$$

and

$$c_P = \frac{1}{R_M C_M a_4}. \quad (21)$$

By using the electric parameters of the piezo actuator the polarisation charge and the leakage current can be determined by means of the equations

$$Q_P(t) = C_P(V(t) - V_M(t)) \quad (22)$$

and

$$I_L(t) = \frac{1}{R_P}(V(t) - V_M(t)). \quad (23)$$

## 4 Measurement results

Figure 4 shows exemplarily the measured signals  $V(t)$ ,  $F_P(t)$ ,  $Q_P(t)$  and  $S_P(t)$  of a piezoelectric actuator which was analysed in the test and measurement device. The signal waveforms of the electric charge  $Q_P(t)$  and the displacement  $S_P(t)$  display the superposition of the electric and mechanical excitation. For the electric excitation a triangular waveform with the frequency  $f_{V0} = 1$  Hz was chosen. The mechanical excitation was performed with a sinusoidal waveform with the frequency  $f_{F0} = 10$  Hz. Generally the choice of the driving signals – i.e. waveform, frequency and range – is mainly restricted by the characteristics of the employed voltage and power sources.

Table 1 lists the parameters of the electromechanical quadripole, calculated on the basis of the illustrated signals.

capacitance $C_P$	14.2 $\mu\text{F}$
leakage resistance $R_P$	330 $\text{M}\Omega$
stiffness $c_P$	100 $\text{N}/\mu\text{m}$
actuator constant $d_A$	0.24 $\mu\text{m}/\text{V}$
sensor constant $d_S$	0.19 $\mu\text{C}/\text{N}$

Table 1: Calculated quadripole parameters

The analysed piezoelectric actuator is a multilayer actuator with the dimensions  $10 \times 10 \times 30 \text{ mm}^3$ . According to the manufacturers description, the actuator reaches a maximum displacement  $S_{\text{max}} = 35 \mu\text{m}$  when a maximum Voltage  $V_{\text{max}} = 200 \text{ V}$  is applied. The small-signal value for the capacitance is denoted as  $C_P = 7 \mu\text{F}$ . As is known, this value can be from 1.5 up to 2 times higher when the actuator is operated in large-signal mode (see Table 1).

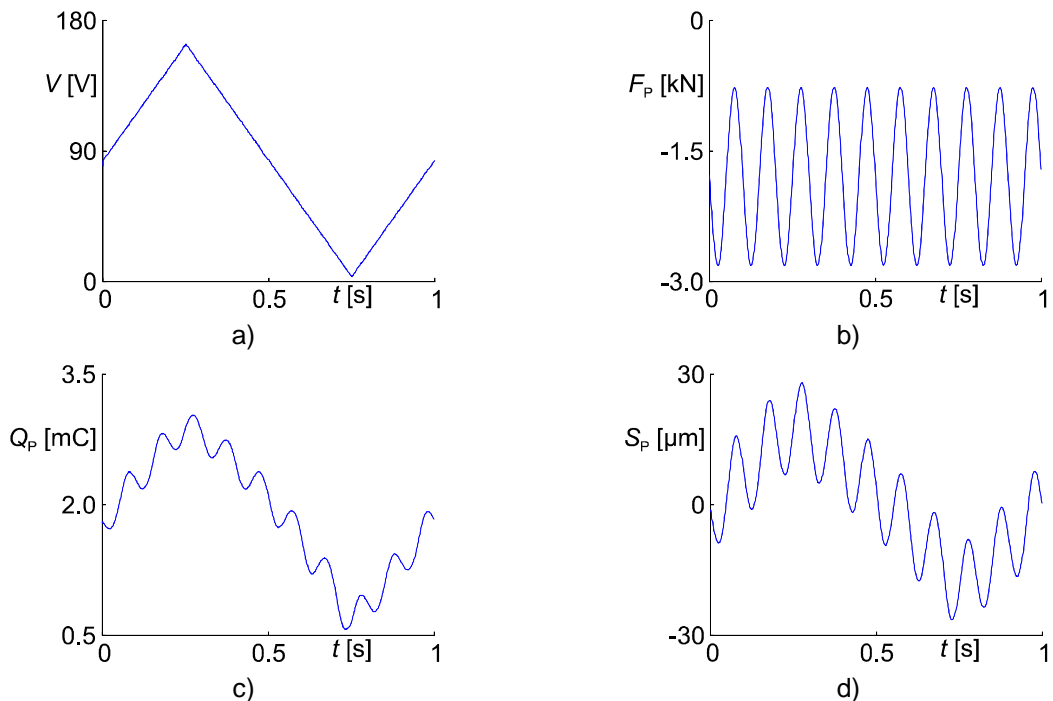


Figure 4: Measured signals a) driving voltage, b) applied load, c) transducer charge, d) displacement

## 5 Summary

The described measuring method facilitates the simultaneous measuring of the electric, mechanical and piezoelectric large-signal properties of piezoelectric actuators during life-cycle testing. These properties are described by the capacitance  $C_p$ , the leakage resistance  $R_p$ , the mechanical stiffness  $c_p$ , the actuator constant  $d_A$  and the sensor constant  $d_S$ . The minimal time required to diagnose a device under test (DUT) equals the maximum of the two given cycle durations  $T_{V0}$  and  $T_{F0}$  of the electric and mechanical control signals. Thus the measuring method can be applied advantageously on a large number of DUTs. The continuously acquired and recorded electric, mechanical and piezoelectric properties allow the monitoring of the DUTs at any time. The presented measuring method has already been implemented in an automatic test and measurement device for life-cycle testing of piezoelectric actuators in large-signal operation [6].

## 6 References:

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