OPERATOR-BASED COMPENSATION OF HYSTERESIS, CREEP AND FORCE-DEPENDENCE OF PIEZOELECTRIC STACK ACTUATORS

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Abstract: The present paper will describe an approach for the simultaneous real-time compensation of hysteresis, creep and force-dependence effects of a piezoelectric stack actuator by a feed-forward controller. The basis of the feed-forward controller is given by so-called complex creep and hysteresis operators, which when adequately implemented, lead to a precise model of the creep and hysteresis characteristics. These complex creep and hysteresis operators consist of the weighted superposition of elementary creep and hysteresis operators which are mathematically simple and which reflect the qualitative properties of the transfer characteristic of the transducer. This operator-based transducer model allows the prediction of transducer displacement resulting from the controller voltage and the reaction force of the surrounded mechanical structure. This information is used within the feed-forward controller in order to calculate the compensation signal. As a result the maximum linearity error caused by hysteresis and creep effects will be lowered by one order of magnitude.

Keywords: compensation, nonlinear control, hysteresis, creep, piezoelectric actuator

1. INTRODUCTION

Piezoelectric stack transducers are capable of immediately transforming electric into mechanical energy or vice versa and are therefore used for industrial purposes as highly dynamic actuators and as fast sensors. Fig. 1 illustrates the conventional operation of a piezoelectric transducer as an actuator.

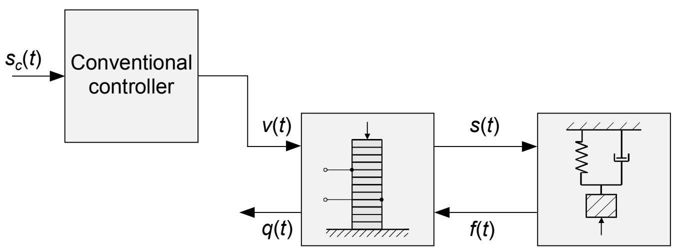

Fig. 1. Conventional operation of a piezoelectric stack actuator

In this case it is driven with an independent voltage $v(t)$ to generate a displacement $s(t)$ against the mechanical surrounding. Beside the voltage-dependent displacement the transducer reacts electrically with a voltage-dependent change of charge $q(t)$. Due to the voltage-dependent displacement the surrounding mechanical structure reacts with a force $f(t)$ against the transducer. This reaction force leads to an additional force-dependent displacement and charge on the electrical contacts of the transducer. Especially when used as an actuator the transducer is driven with high voltage amplitudes to generate the largest possible displacements. In this electrical large-signal operation the electromechanical behaviour shows strong creep and hysteresis effects (Kuhnen and Janocha, 1998). In the case of small force amplitudes the characteristic of the piezoelectric stack transducer can be divided into a creep and hysteretic voltage-dependent part described here by the general scalar operator T and a weighted linear force-dependent part characterised by the so-called small-signal elasticity S.

$$s(t) = T[v](t) + S\,f(t) \tag{1}$$

The hysteresis and creep effects in the voltage-dependent part on the one side and the linear force-dependence on the other side leads to ambiguities in the characteristic of piezoelectric transducers and thus to a considerable reduction of the repeatability attainable in open loop control. One possible solution of this problem is to compensate the hysteresis, creep and force-dependence simultaneously using the inverse feed-forward compensator

$$v(t) = T^{-1}[s_{\mathrm{c}} - S\,f_m](t), \tag{2}$$

see Fig. 2. In this equation $s_c(t)$ is the desired diplacement and $f_m(t)$ the force measured by a force sensor. The force sensor can be realized for example if we use one disk of the actuator as a sensor element. For small-signal forces the relationship between the electrical charge and the force is given by the piezoelectric effect

$$q(t) = d\,f(t) \tag{3}$$

for this disc. In this case the force sensor consists of the sensor disc of the actuator and a conventional charge sensor electronic (Tichy and Gautschi, 1980).

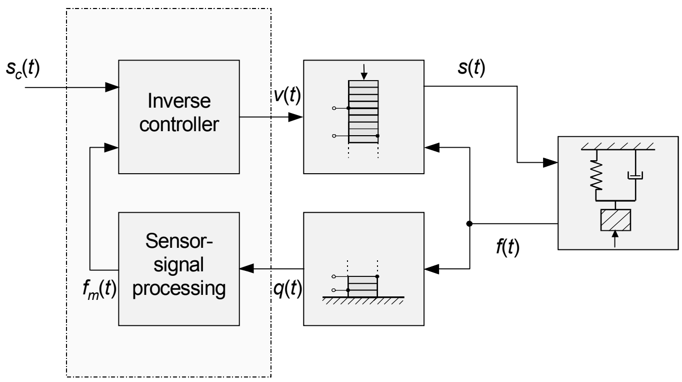

Fig. 2. Inverse control of a piezoelectric stack actuator

2. HYSTERSIS AND CREEP MODELLING

In the mathematical literature the notation of the hysteretic nonlinearity will be equated with the notation "rate independent memory effect" (Brokate and Sprekels, 1996, Visintin, 1996). This means that the output signal of a system with hysteresis depends not only on the present value of the input signal but also on the order of their amplitudes, especially their extremum values, but not on their rate in the past. Because of its phenomenological character the concept of hysteresis operators developed by Krasnosel'skii and Pokrovskii in the 1970's allows a very general and precise modelling of hysteretic system characteristics (Krasnosel'skii and Pokrovskii, 1989). The basic idea consists of the modelling of the real hysteretic transfer characteristic by the weighted superposition of many elementary hysteresis operators, which differ in terms of one or more parameters depending on the type of the elementary operator. One type of such an elementary hysteresis operator is the so-called play operator

$$z_r(t) = p_r[v, z_{r0}](t) \tag{4}$$

which is defined by the recursive equations

$$z_r(t) = P(v(t), z_r(t_i), r) \tag{5}$$

and

$$z_r(t_0) = P(v(t_0), z_{r0}, r) \tag{6}$$

with

$$P(v,z,r) = \max\{v-r, \min\{v+r,z\}\} \qquad (7)$$

for piecewise monotonous input signals with a monotonicity partition $t_0 \leq t_1 \leq \ldots \leq t_i \leq t \leq t_{i+1} \ldots \leq t_N$. The operator is characterized by its threshhold parameter r. The initial value of the operator state z_{r0} determines in a clear manner the value of the operator output $z_r(t)$ in dependence of the future values of the input signal $v(t)$. Fig. 3 shows the rate-independent output-input trajectory of this simple hysteresis operator.

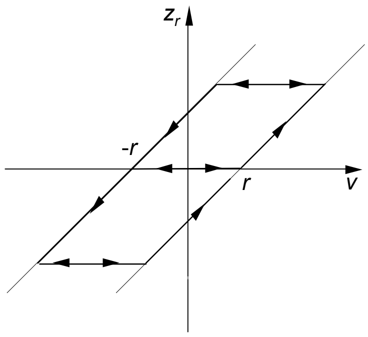

Fig. 3. Rate-independent transfer characteristic of the play operator

For the precise modelling of real hysteresis phenomena more play operators with different threshold values r_i can be superimposed. This parallel connection of elementary hysteresis operators leads to the complex hysteresis operator

$$H[v](t) = \sum_{i=1}^{n} b_i \, p_{r_i}[v, z_{r_i 0}](t). \qquad (8)$$

In practice due to the continuity of the play operator complex hysteresis loops can be modeled with sufficient precision with the help of a small number of elementary operators (Bergqvist, 1994). Therefore the hysteresis operator (8) is a suitable tool for the real-time calculation of the complex hysteretic behaviour.

The notion of creep originates from the field of solid-mechanics and describes the time-variant deformation behaviour of a body due to a sudden mechanical load (Kortendieck, 1993, Lemaitre and Chaboche, 1990). It is a strongly damped, rate-dependent phenomenon, which can be found in a similar manner in the field of ferromagnetism and ferroelectricity. Like hysteresis phenomena electrically induced creep effects have a considerable influence on the large-signal transfer characteristic of a piezoelectric transducer (Kuhnen and Janocha, 1998). In a first order approximation these creep phenomena are presumed linear. As a consequence they can be described, analogously to the hysteresis modelling process, by a complex linear creep operator

$$L[v](t) = \sum_{j=1}^{m} c_j \, l_{a_j}[v, z_{a_j 0}](t), \qquad (9)$$

given by a weighted superposition of many elementary linear creep operators with different creep eigenvalues a. In this case the elementary linear creep operator is the solution operator

$$l_a[v, z_{a0}](t) = \mathrm{e}^{-a(t-t_0)} z_{a0} + a\int_{t_0}^{t} \mathrm{e}^{a(\tau - t)} v(\tau)\, \mathrm{d}\tau \qquad (10)$$

of a linear, first order differential equation with an initial value z_{a0}. Fig. 4 shows the step response of the elementary linear creep operator, which has the same qualitative features as the step response of the creep phenomena in the real system.

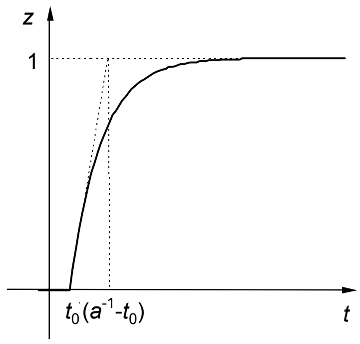

Fig. 4. Step response of a rate dependent, elementary linear creep operator

Analogous to the complex hysteresis operator, creep curves can be modeled with sufficient precision with the help of a small number of elementary operators (Kuhnen and Janocha, 1998). So the creep operator (9) is also a suitable tool for the real-time calculation of complex linear creep effects.

In the following we derive a first-order approximation model of the actuatoric characteristic T by the linear superposition of a weighted reversible part, a rate-independent irreversible part described by the complex hysteresis operator H and a rate-dependent part described by the complex linear creep operator L.

$$T[v](t) = d\, v(t) + H[v](t) + L[v](t) \qquad (11)$$

In electrical small signal operation (11) can be reduced to the reversible part

$$T[v](t) = d\, v(t) \qquad (12)$$

with the piezoelectric constant d. From this follows that the operator-based approach is a logical extension of small-signal modelling to the large-signal range.

3. TIME-DISCRETE COMPENSATOR FOR REAL-TIME APPLICATIONS

To calculate the compensation signal by the inverse operator T^{-1} in real-time a digital signal processor (DSP) is used. Therefore a time-discrete model for the operator T is developed. Using a rectangular approximation for the numerical calculation of the integral equation (10), we obtain a simple first-order difference equation

$$z_a(k) = L(v(k-1), z_a(k-1), a) \qquad (13)$$

with

$$L(v,z,a) = \mathrm{e}^{-aT_s} z + (1-\mathrm{e}^{-aT_s})\, v \qquad (14)$$

and with the initial value $z_a(0) = z_{a0}$. In equation (14) T_s is the sampling time.

A procedure for the numerical calculation of the play operator and thus a time-discrete model for digital signal processing applications follows from equation (5) for the time-continuous counterpart. The time-discrete play operator can be calculated according to the difference equation

$$z_r(k) = P(v(k), z_r(k\text{-}1), r) \qquad (15)$$

and

$$z_r(0) = P(v(0), z_{r0}, r). \qquad (16)$$

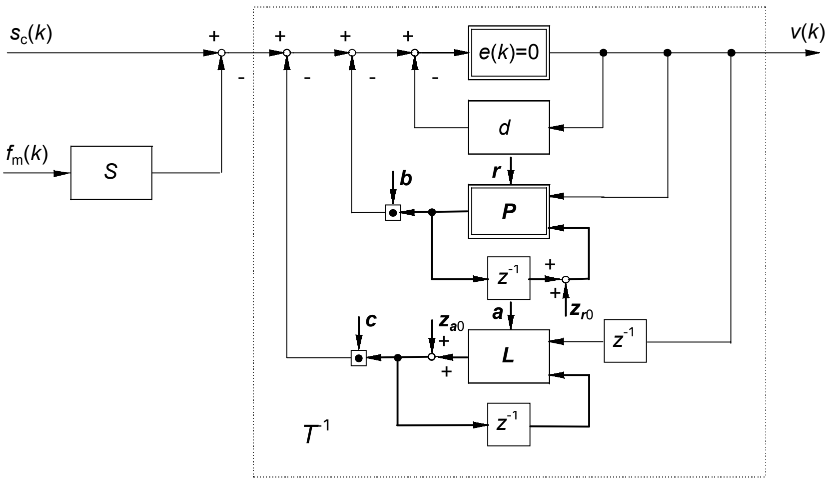

Fig 5. Signal flow chart of the inverse controller

The time-discrete counterpart of (1) is then given by the solution of the system of first-order difference equations

$$s(k) = d\, v(k) + \boldsymbol{b}^T \cdot \boldsymbol{z_r}(k) + \boldsymbol{c}^T \cdot \boldsymbol{z_a}(k) + S f(k) \qquad (17)$$

$$\boldsymbol{z_r}(k) = \boldsymbol{P}(v(k), \boldsymbol{z_r}(k-1), \boldsymbol{r}) \qquad (18)$$

$$\boldsymbol{z_a}(k) = \boldsymbol{L}(v(k-1), \boldsymbol{z_a}(k-1), \boldsymbol{a}) \qquad (19)$$

with the initial values

$$\boldsymbol{z_r}(0) = \boldsymbol{P}(v(k), \boldsymbol{z}_{r0}, \boldsymbol{r}) \qquad (20)$$

$$\boldsymbol{z_a}(0) = \boldsymbol{z}_{a0} \qquad (21)$$

and the vectors

$$\boldsymbol{r}^T = (r_1 \,..\, r_n),\ \boldsymbol{a}^T = (a_1 \,..\, a_m),$$
$$\boldsymbol{b}^T = (b_1 \,..\, b_n),\ \boldsymbol{c}^T = (c_1 \,..\, c_m),$$
$$\boldsymbol{z_r}^T = (z_{r_1} \,..\, z_{r_n}),\ \boldsymbol{z_a}^T = (z_{a_1} \,..\, z_{a_m}),$$
$$\boldsymbol{P}(v,\boldsymbol{z},\boldsymbol{r})^T = (P(v,z_{r_1},r_1) \,..\, P(v,z_{r_n},r_n)),$$
$$\boldsymbol{L}(v,\boldsymbol{z},\boldsymbol{a})^T = (L(v,z_{a_1},a_1) \,..\, L(v,z_{a_m},a_m)) .$$

The problem to find the inverse control value $v(k)$ for a given control value $s_c(k)$ and a given measurement value $f_m(k)$ is equivalent to the solution of the implicit time-discrete difference equation

$$\begin{aligned} e(v(k)) &= s_c(k) - d\, v(k) - \boldsymbol{b}^T \cdot \boldsymbol{P}(v(k), \boldsymbol{z_r}(k\text{-}1), \boldsymbol{r}) \\ &\quad -\boldsymbol{c}^T \cdot \boldsymbol{L}(v(k-1), \boldsymbol{z_a}(k-1), \boldsymbol{a}) - S f_m(k) \\ &= 0 \end{aligned} \qquad (22)$$

with the implicit initial value equation

$$\begin{aligned} e(v(0)) &= s_c(0) - d\, v(0) - \boldsymbol{b}^T \cdot \boldsymbol{P}(v(0), \boldsymbol{z}_{r0}, \boldsymbol{r}) \\ &\quad -\boldsymbol{c}^T \cdot \boldsymbol{z}_{a0} - S f_m(0) \\ &= 0 \end{aligned} \quad . \qquad (23)$$

If the coefficients d and $\boldsymbol{b}$ fulfill the contraints $d > 0$ and $\boldsymbol{b} \geq \boldsymbol{0}$, the continuity and monotonicity properties of the play operator lead to a continuous and strictly monotone relationship between the value $e(v(k))$ and the value $v(k)$ for every feasible state value $\boldsymbol{z_r}(k\text{-}1)$. Therefore there exists only one value $v(k)$ and only one feasible subsequent state value $\boldsymbol{z_r}(k)$ which fulfills

(22) and thus the zero finding problem in the k-th time point has only one global solution which can be calculated numerically by a bisection method. After the numerical solution of (22) the present state values $z_r(k)$ of the hysteresis operator and $z_a(k)$ of the creep operator can be calculated according to (18) and (19). Fig. 5 shows the feedback principle of the inverse controller.

4. RESULTS AND DISCUSSION

The inverse compensator was realized on a digital signal processor (DSP) with a sampling rate of 1 kHz. To verify the performance of the compensation concept the inverse compensator was driven with the desired displacement signal $s_c(t)$ shown in Fig. 6a. Fig. 6c shows the characteristic of the conventional linear controller as a gray line.

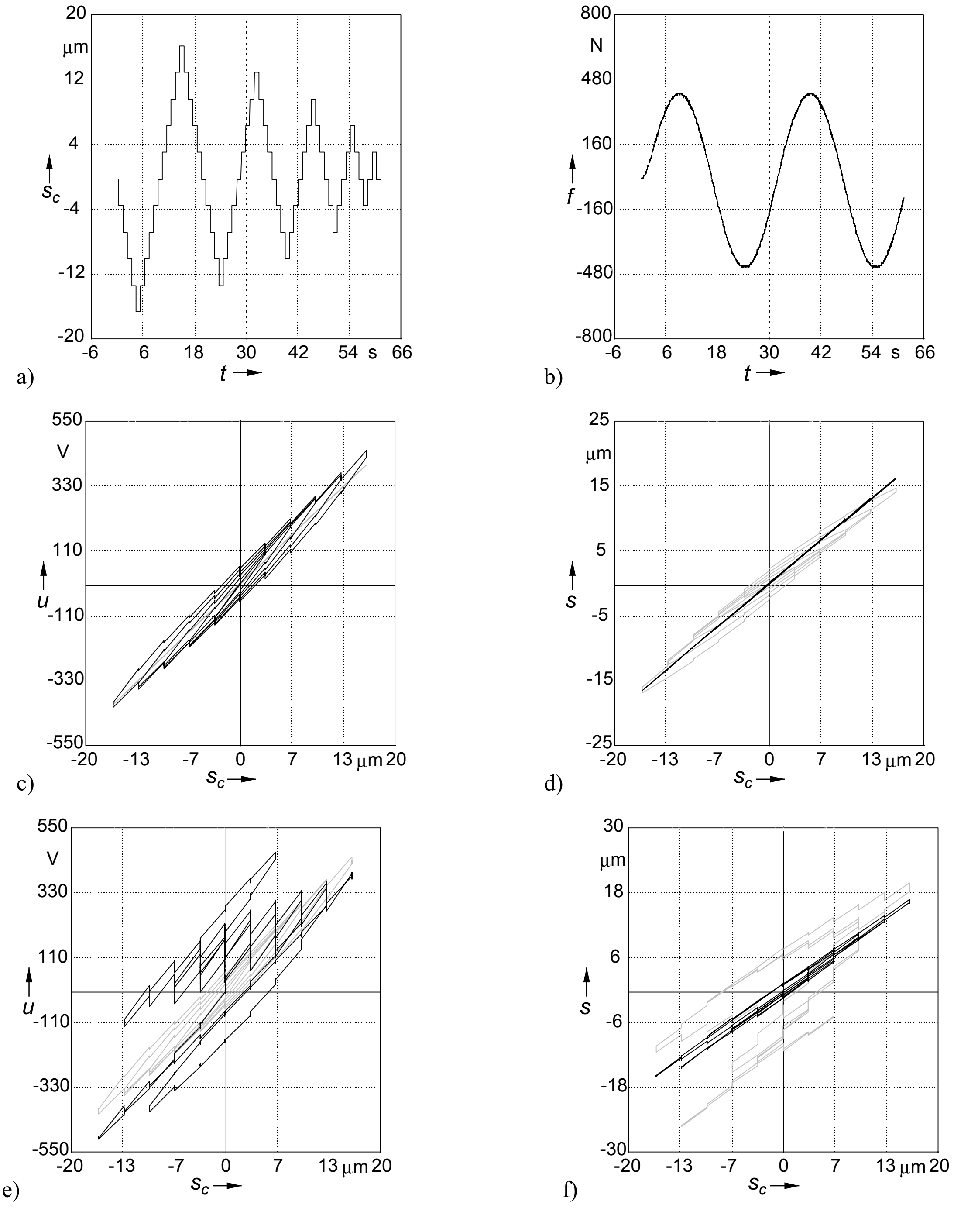

Fig. 6. Hysteresis, creep and force-dependence compensation results

It is an ideal linear rate-independent characteristic typical for conventional voltage-amplifiers. As a consequence the characteristic of the serial combination conventional controller-transducer, shown in Fig. 6d as a gray line, shows the hysteresis and creep effects of the transducer. The characteristic of the operator-based inverse compensator, shown in Fig. 6c as a black line, is obviously inverse to the characteristic of the transducer. As a consequence the characteristic of the serial combination inverse compensator-transducer, shown in Fig. 6d as a black line, is almost completely free of hysteresis and creep effects and the displacement error caused by creep and hysteresis effects is reduced from 2.47 µm using the conventional controller to 0.25 µm using the inverse compensator. This is an improvement of one order of magnitude.

The gray line in Fig. 6f shows the strongly disturbed characteristic of the serial combination inverse compensator-transducer. The disturbance of the characteristic is generated by the additional external force signal shown in Fig. 6b. In this case the force-dependent part of the displacement is not compensated by the inverse controller, see the gray line in Fig. 6e. This force-dependence effect leads to a displacement error of 11.8 µm. The black line in Fig. 6e shows the strongly disturbed characteristic of the inverse controller. The disturbance of the inverse hysteretic and creep characteristic is caused by an additional compensation of displacement generated by the external force signal. As a consequence the force generated disturbances in the characteristic of the serial combination inverse compensator-transducer is strongly reduced, see the black line in Fig. 6f. As the main result the displacement error caused by the external force is reduced to 1.55 µm. This is also an improvement of one order of magnitude.

5. SUMMARY AND PROSPECTS

This paper has shown that complex creep and hysteresis operators offer an efficient method to model the electromechanical characteristic of a piezoelectric stack actuator if it is driven electrically in the large-signal range. Based on this method a compensator for the simultaneous compensation of hysteresis and creep effects in real-time was presented. Beside hysteresis and creep phenomena which can be regarded as intrinsic disturbances the displacement signal of the transducer is also disturbed by the reaction force of the mechanical surrounding of the actuator. The extension of the inverse feed-forward controller to compensate this force-dependence was also presented for the case of small signal reaction forces.

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