INVERSE FEEDFORWARD CONTROLLER FOR COMPLEX HYSTERETIC NONLINEARITIES IN SMART-MATERIAL SYSTEMS

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ABSTRACT

Undesired complex hysteretic nonlinearities are present to varying degree in virtually all smart material based sensors and actuators provided that they are driven with sufficient high amplitudes. This necessitates the development of purely phenomenological models which characterize these nonlinearities in a way which is sufficiently accurate, robust, amenable to control design for nonlinearity compensation and efficient enough for use in real-time applications. To fulfill these demanding requirements the present paper describes a new compensator design method for invertible complex hysteretic nonlinearities which bases on the so-called Prandtl-Ishlinskii hysteresis operator. The parameter identification of this model can be formulated as a quadratic optimization problem which produces the best L_2^2-norm approximation for the measured output-input data of the real hysteretic nonlinearity. Special linear inequality constraints for the parameters guarantee the unique solvability of the identification problem and the invertability of the identified model. This leads to a robustness of the identification procedure against unknown measurement errors, unknown model errors and unknown model orders. The corresponding compensator can be directly calculated and thus efficiently implemented from the model by analytical transformation laws. Finally the compensator design method is used to generate an inverse feedforward controller for a magnetostrictive actuator. In comparision to the conventional controlled magnetostrictive actuator the nonlinearity error of the inverse controlled magnetostrictive actuator is lowered from about 30 % to about 3 %.

KEYWORDS

Hysteresis, Nonlinear Systems, Modeling, Identification, Inverse Controller

INTRODUCTION

Complex memory-free nonlinearities or in generalization complex hysteretic nonlinearities are present to varying degree in virtually all smart material based sensors and actuators provided that they are driven with sufficient high amplitudes. Well-known complex hysteretic nonlinearities are the magnetic induction - magnetic field relation of ferromagnetic materials, the electrical polarization - electrical field relation of ferroelectric materials and the stress - strain relation of elasto-plastic materials. The most familiar examples for complex hysteretic nonlinearities in smart material systems are piezoelectric, magnetostrictive and shape memory alloy based actuators and sensors [1]. In many applications, these nonlinearities can be limited through the choice of proper materials and operating regimes so that linear sensor and actuator characteristics can be assumed. In the consequence of more stringent performance requirements a large number of systems are currently operated in regimes in which hysteretic nonlinearities are unavoidable. This necessitates the development of purely phenomenological models which characterize these nonlinearities in a way which is sufficiently accurate, robust, amenable to control design for nonlinearity compensation and efficient enough for use in real-time applications.

Models of hysteretic nonlinearities have evolved from two different branches of physics: ferromagnetism and plasticity theory. The roots of both branches go back to the end of the 19th century. But only at the beginning of the seventies of the 20th century was a mathematical formalism for a systematic consideration of hysteretic nonlinearities developed [2]. The core of this theory is formed by so-called hysteresis operators which describe hysteretic transducers as a mapping between function spaces. But it is only since the beginning of the 90s that engineers employ this theory on a larger scale to develop modern strategies for the linearisation of hysteretic systems with an inverse control approach. Whereas in the beginning mainly the well-known Preisach operator was used for the modeling and linearization of solid-state actuators with the inverse control approach [3,4], recent papers also reference the so-called Prandtl-Ishlinskii operator [5,6,7] which belongs to an important subclass of the Preisach operator [8].

To develop a consistent phenomenological design concept for a compensator of an invertable complex hysteretic nonlinearity which is sufficiently flexible in its modeling capabilities and moreover well suited for real-time applications is not a simple task because it covers in general the following coupled design steps: modeling the real hysteretic nonlinearity, identification of the model parameters to adapt the model to the real hysteretic nonlinearity and inversion of the model to obtain the desired compensator. Especially the mathematical complexity of the identification and inversion problem depends on the phenomenological modeling method (for example Preisach or Prandtl-Ishlinskii modeling) and influences strongly the practical use of the design concept. Another difficulty of the identification problem follows from the strong sensitivity of the model parameters to unknown measurement errors of the output-input data, unknown model errors and unknown model orders. Due to these effects a parameter identification can result in the best case to a poor model accuracy or in the worst case to a locally non-invertable model and as a consequence the whole compensator design fails. Therefore the robustness against these effects is an inherent requirement for a consistent phenomenological compensator design method. To overcome these difficulties the present paper describes a new compensator design concept for complex hysteretic nonlinearities based on the Prandtl-Ishlinskii modeling approach which is robust in the sense mentioned above. The robustness of the new compensator design method is reached by the consideration of linear inequality constraints for the free model parameters which guarantee a search for the best $L_2{}^2$-norm approximation of the measured output-input data only in those parameter ranges where the identified model is invertible.

HYSTERESIS MODELING

In the mathematical literature the notation of the hysteretic nonlinearity will be equated with the notation "rate-independent memory effect" [8,9,10]. At the beginning of the 20th century Madelung investigated experimentally the branchings and loopings of ferromagnetic hysteresis which result from the rate-independent memory property and stated the following three rules from his observations [8], see figure 1.

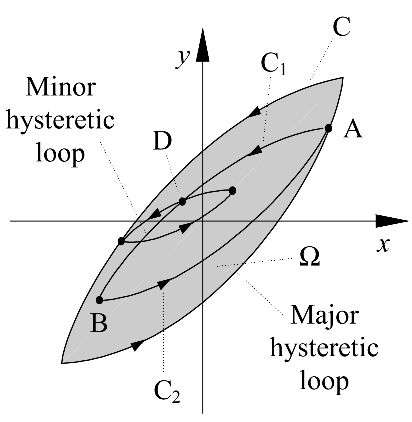

Figure 1: Complex hysteretic nonlinearity

1. Any curve C_1 emanating from a turning point A of the output-input trajectory is uniquely determined by the coordinates of A.
2. If any point B on the curve C_1 becomes a new turning point, then the curve C_2 originating at B leads back to the point A.
3. If the curve C_2 is continued beyond the point A, then it coincides with the continuation of the curve C which led to the point A before the C_1-C_2 - cycle was traversed.

In addition to these three Madelung's rules a fourth important observation can be made for ferromagnetic, ferroelectric, elasto-plastic materials and actuator and sensor characteristics of smart materials, and it is exactly this property of real hysteretic nonlinearities in which the complex ones differ from the non complex ones.

4. From a non turning point D within the hysteretic region Ω more than one branch can be traversed.

Because of its phenomenological character the concept of hysteresis operators allows a powerful modelling of complex hysteretic nonlinearities without taking into account the underlying physics [2]. The basic idea consists of the modeling of the real complex hysteretic nonlinearities by the weighted superposition of many so-called elementary hysteresis operators. Elementary hysteresis operators are non complex hysteretic nonlinearities with a simple mathematical structure which are characterized by one or more parameters. One of the most familiar and most important elementary hysteretic mapping

$$y(t) = H_r[x, y_0](t) \tag{1}$$

between the input signal x and the output signal y is the so called play or backlash operator H_r which is often used to model mechanical play in gears with one degree of freedom. It is normally defined by the recursive equation

$$y(t) = H(x(t), y(t_i), r) \tag{2}$$

with the initial condition

$$y(t_0) = H(x(t_0), y_0, r) \tag{3}$$

for the output signal at initial time t_0. It dependends on the independent initial value y_0 of the output and the sliding symmetrical dead-zone function

$$H(x, y, r) = \max\{x - r, \min\{x + r, y\}\} \tag{4}$$

for piecewise monotonous input signals with a monotonicity partition $t_0 \leq t_1 \leq . . \leq t_i \leq t \leq t_{i+1} . . \leq t_N = t_e$ [8]. The operator is characterized by its threshold parameter $r \in \Re^+_0$. Figure 2 shows the rate-independent output-input trajectory of this elementary hysteresis operator. Although the three Madelung's rules hold for the play operator it can be easily realized that the ferromagnetic, ferroelectric or elastic-plastic behaviour of real materials and the hysteretic actuator and sensor characteristics of real smart materials are of much higher complexity, note also rule 4.

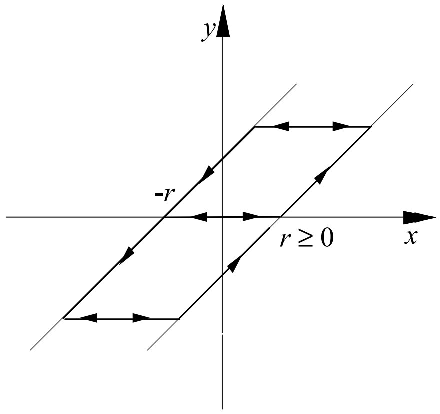

Figure 2: Rate-independent characteristic of H_r

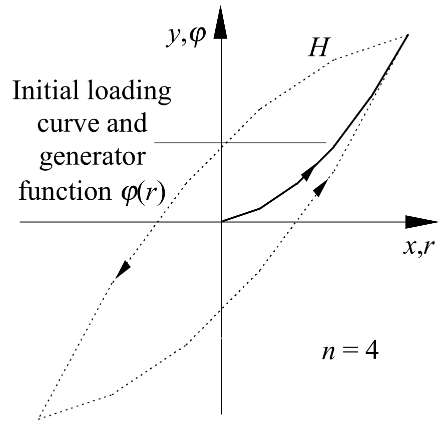

Figure 3: Initial loading curve and generator function $\varphi(r)$

To obtain a more powerful model for complex hysteretic nonlinearities we introduce the so-called Prandtl-Ishlinskii hysteresis operator H by the linear weighted superposition of many play operators with different threshold values. From this follows

$$H[x](t) := \boldsymbol{w}^{T} \cdot \boldsymbol{H_r}[x, \boldsymbol{z}_0](t) \quad (5)$$

with the vector of weights $\boldsymbol{w}^T = (w_0\ w_1\ ..\ w_n)$, the vector of thresholds $\boldsymbol{r}^T = (r_0\ r_1\ ..\ r_n)$ with $0 = r_0 < r_1 < .. < r_n < +\infty$, the vector of the initial states $\boldsymbol{z}_0^T = (z_{00}\ z_{01}\ ..\ z_{0n})$ of the play operators and the vector of the play operators

$$\boldsymbol{H_r}[x, \boldsymbol{z}_0](t)^T = (H_{r_0}[x, z_{00}](t)\ .\ .H_{r_n}[x, z_{0n}](t)).$$

The hysteretic characteristic of the Prandtl-Ishlinskii hysteresis operator is completely defined by the characteristic of the so-called initial loading curve. This special branch will be traversed if the initial state of the Prandtl-Ishlinskii hysteresis operator is zero and it is driven with a monotonous increasing input signal. The initial loading curve can be fully characterized by and therefore equated with a threshold-dependent piecewise linear function

$$\varphi(r) = \sum_{j=0}^{i} w_j (r - r_j)\ ;\ r_i \le r < r_{i+1}\ ;\ i = 0\ .\ .\ n\,, \quad (6)$$

with $r_{n+1} = \infty$ and

$$\frac{\mathrm{d}}{\mathrm{d}r}\varphi(r) = \sum_{j=0}^{i} w_j\ ;\ r_i \le r < r_{i+1}\ ;\ i = 0\ .\ .\ n\,. \quad (7)$$

It is called the generator function of the Prandtl-Ishlinskii hysteresis operator [11], see figure 3 for a Prandtl-Ishlinskii hysteresis operator with a model order of $n = 4$.

HYSTERESIS COMPENSATION

Under the consideration of the linear inequalitiy constraints

$$\boldsymbol{U} \cdot \boldsymbol{w} - \boldsymbol{u} \le \boldsymbol{0} \quad (8)$$

for the weights with the matrix

$$\boldsymbol{U} = \begin{pmatrix} -1 & 0 & . & . & 0 \\ -1 & -1 & . & . & 0 \\ . & . & . & & . \\ . & . & & . & . \\ -1 & -1 & . & . & -1 \end{pmatrix}, \text{ the vector } \boldsymbol{u} = \begin{pmatrix} -\varepsilon \\ -\varepsilon \\ . \\ . \\ -\varepsilon \end{pmatrix} \text{ and a}$$

possibly infinite small number $\varepsilon > 0$ the generator function is strongly monotonous for $r \ge 0$ and therefore the inverse of the generator function $\varphi^{-1}(r)$ exists uniquely for $r \ge 0$. $\varphi^{-1}(r)$ is piecewise linear and strongly monotonous and can therefore also be regarded as a generator function

$$\varphi'(r') = \sum_{j=0}^{i} w'_j (r' - r'_j)\ ;\ r'_i \le r' < r'_{i+1}\ ;\ i = 0\ .\ .\ n\,, \quad (9)$$

of a Prandtl-Ishlinskii hysteresis operator with $r_{Hn+1}' = \infty$ and

$$\frac{\mathrm{d}}{\mathrm{d}r'}\varphi(r') = \sum_{j=0}^{i} w'_j\ ;\ r'_i \le r' < r'_{i+1}\ ;\ i = 0\ .\ .\ n\,, \quad (10)$$

namely the inverse Prandtl-Ishlinskii hysteresis operator

$$H^{-1}[y](t) := \boldsymbol{w}'^{T} \cdot \boldsymbol{H}_{\boldsymbol{r}'}[y, z'_0](t) \quad (11)$$

with transformed initial states z_0', threshold values $\boldsymbol{r}'$ and weights $\boldsymbol{w}'$. In this case the weights fullfil the same linear inequaltity constraints

$$\boldsymbol{U} \cdot \boldsymbol{w}' - \boldsymbol{u} \leq \boldsymbol{0} \, . \quad (12)$$

The transformation law $\boldsymbol{r}' = \boldsymbol{\Omega}(\boldsymbol{r}, \boldsymbol{w})$ for the thresholds results from the relation $r_i' = \varphi(r_i)$. From this follows

$$r_i' = \sum_{j=0}^{i} w_j (r_i - r_j) \quad ; \quad i = 0 \, . \, . \, n \quad (13)$$

for the threshold-discrete case, see figure 4.

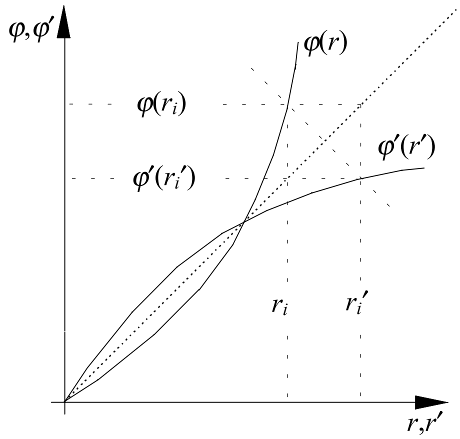

Figure 4: Generator functions $\varphi(r)$ and $\varphi'(r')$

The transformation law $\boldsymbol{w}' = \boldsymbol{\Xi}(\boldsymbol{w})$ for the weights results from the relation $\mathrm{d}\varphi(r_i')/\mathrm{d}r' = 1/(\mathrm{d}\varphi(r_i)/\mathrm{d}r)$, see figure 4. From this follows

$$w_0' = \frac{1}{w_0} \qquad \text{and}$$

$$w_i' = -\frac{w_i}{(w_0 + \sum_{j=1}^{i} w_j)(w_0 + \sum_{j=1}^{i-1} w_j)} \quad ; \quad i = 1 \, . \, . \, n \, . \quad (14)$$

The transformation law $\boldsymbol{z}_0' = \boldsymbol{\Psi}(\boldsymbol{z}_0, \boldsymbol{w})$ for the initial states results from the relation

$$(z_{0i+1}' - z_{0i}')/(r_{i+1}' - r_i') = (z_{0i+1} - z_{0i})/(r_{i+1} - r_i)$$

which is the threshold-discrete counterpart to the relation $\mathrm{d}z(r')/\mathrm{d}r' = \mathrm{d}z(r)/\mathrm{d}r$ for the threshold-continuous case discussed in [11]. From this follows the transformation law

$$z_{0i}' = \sum_{j=0}^{i} w_j z_{0i} + \sum_{j=i+1}^{n} w_j z_{0j} \quad ; \quad i = 0 \, . \, . \, n \quad (15)$$

for the initial states. The Prandtl-Ishlinskii hysteresis operator has the following more or less obvious properties:

1. Because the Madelung's rules persist under linear superposition, they hold also for the Prandtl-Ishlinskii hysteresis operator. Moreover due to the $n > 1$ inner hysteretic state variables different branches can be traversed from a non turning point D which is in agreement with rule 4.
2. The closed loops which will be traversed for input signals oscillating between maximum and minimum values have an odd symmetry to the center point of the corresponding loop. This odd symmetry property is a property of the play operator and persists also under linear superposition.
3. The inversion operation which is given by the transformation laws does not change the structure of the Prandtl-Ishlinskii hysteresis operator and its inequality contraints for the weights.

Property 1 agrees at least qualitatively with experimental observations for complex hysteretic nonlinearities. Property 3 leads to a direct formulation and thus to a very efficient implementation of the corresponding compensator which is profitable for real-time control applications. The odd symmetry property 2 which is an inherent model characteristic is the main drawback of this Prandtl-Ishlinskii modeling approach. But in many practical cases this property is often fulfilled. Well-known examples are piezoelectric and magnetostrictive actuators driven in operating regimes with moderate input amplitudes.

HYSTERESIS IDENTIFICATION

The identification procedure which is used to adapt the model to the real hysteretic nonlinearity is divided into two parts. In the first part the thresholds $\boldsymbol{r}$ of the Prandtl-Ishlinskii hysteresis operator are determined by the formula

$$r_i = \frac{i}{n+1} \max_{t_0 \leq t \leq t_e} \{|x(t)|\} \quad ; \quad i = 0 \, . \, . \, n \, . \quad (16)$$

The identification of the weights $\boldsymbol{w}$ of the Prandtl-Ishlinskii hysteresis operator which is the object of the second part can be formulated as an $L_2{}^2$-norm minimization of the so-called output error model

$$E[x, y](t) := \boldsymbol{w}^T \cdot \boldsymbol{H_r}[x, \boldsymbol{z}_0](t) - y(t) \quad (17)$$

which is linear dependent on the weights. This leads to the quadratic optimization problem

$$\min_{\boldsymbol{w} \in \Re^{n+1}} \{\boldsymbol{w}^T \cdot \int_{t_0}^{t_e} \boldsymbol{H_r}[x, \boldsymbol{z}_0](t) \boldsymbol{H_r}[x, \boldsymbol{z}_0]](t)^T \, \mathrm{d}t \cdot \boldsymbol{w} - \int_{t_0}^{t_e} 2 y(t) \boldsymbol{H_r}[x, \boldsymbol{z}_0]](t)^T \, \mathrm{d}t \cdot \boldsymbol{w} + \int_{t_0}^{t_e} y(t)^2 \, \mathrm{d}t\} \quad (18)$$

with the linear inequality contraints

$$\boldsymbol{U} \cdot \boldsymbol{w} - \boldsymbol{u} \leq \boldsymbol{0} \tag{19}$$

which has one global solution and which ensures the invertability of the identified Prandtl-Ishlinskii hysteresis operator. This guarantees a unique best L_2^2-norm approximation of the measured hysteretic characteristic in that space of the weights which leads to an invertible Prandtl-Ishlinskii hysteresis operator. Therefore the invertability of the Prandtl-Ishlinskii hysteresis operator and its inverse is always guaranteed during the optimization and thus the design process for the model and the corresponding compensator is consistent and robust against unknown measurement errors of input-output data, unknown model errors and unknown model orders.

RESULTS

In this section the performance of the presented compensator design method for complex hysteretic nonlinearities will now be demonstrated by means of the displacement-current relation of a magnetostrictive transducer. Figure 5 shows the strongly monotonous hysteretic displacement-current relation in the moderate-signal operating range of the magnetostrictive actuator.

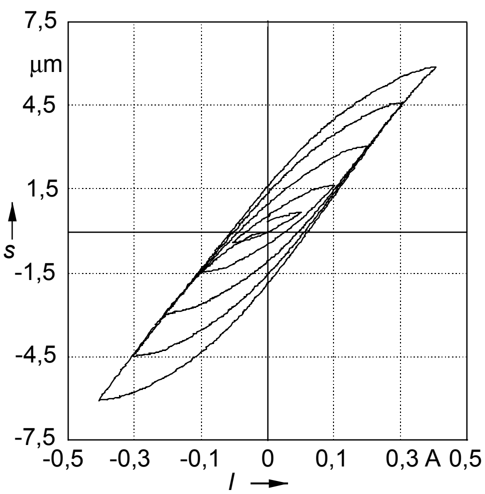

Figure 5: Measured hysteretic displacement-current relation

It is mainly characterized by strongly monotonous branches and symmetrical hysteretic loops with a counterclockwise orientation. Therefore the modeling, identification and compensation of this real complex hysteretic nonlinearity can be realized with the Prandtl-Ishlinskii approach. The model order n = 0 leads to a linear rate-independent operator model and thus the identification procedure determines the best linear L_2^2-norm approximation of the real hysteretic nonlinearity. The nonlinearity error defined by

$$\frac{\max\limits_{t_0 \leq t \leq t_e} \{|H[I](t) - s(t)|\}}{\max\limits_{t_0 \leq t \leq t_e} \{|H[I](t)|\}} \tag{20}$$

amounts in this case up to 29,8 %. Figure 6 shows the looping and branching behaviour of the Prandtl-Ishlinskii hysteresis operator with a model order of n = 14 as a result of the identification procedure.

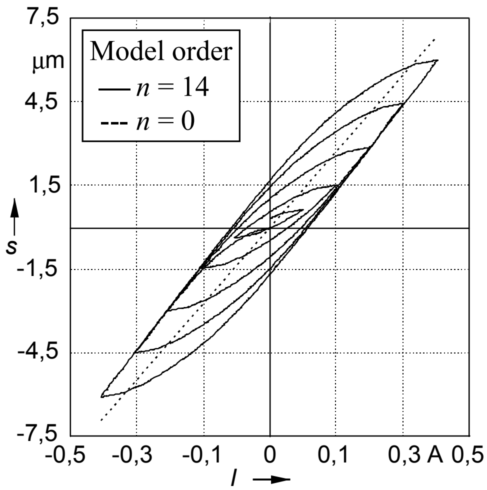

Figure 6: Modeled hysteretic displacement-current relation

The nonlinearity error amounts in this case to 3,0 % which is nearly ten times smaller as for the best linear L_2^2-norm approximation. Due to unknown model errors a further increasing of the model order doesn't improve the nonlinearity error in this case.

For the compensation of the real hysteretic nonlinearity a feedforward controller is used which bases on the inverse Prandtl-Ishlinskii hysteresis operator, see figure 7. $s_c(t)$ is the given displacement signal value. The inverse Prandtl-Ishlinskii hysteresis operator is obtained from the Prandtl-Ishlinskii hysteresis operator using the transformation laws for the thresholds (13), the weights (14) and the initial states (15).

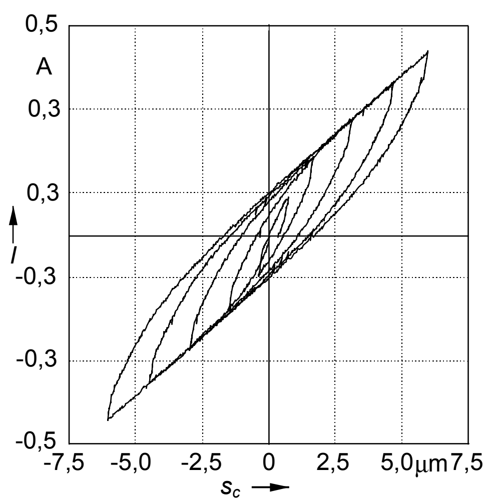

Figure 7: Inverted hysteretic displacement-current relation

It is realized by a digital signal processor with a sampling rate of up to 10 kHz and a displacement controlled current source. The looping and branching characteristic of the inverse Prandtl-Ishlinskii hysteresis operator is shown in figure 7.

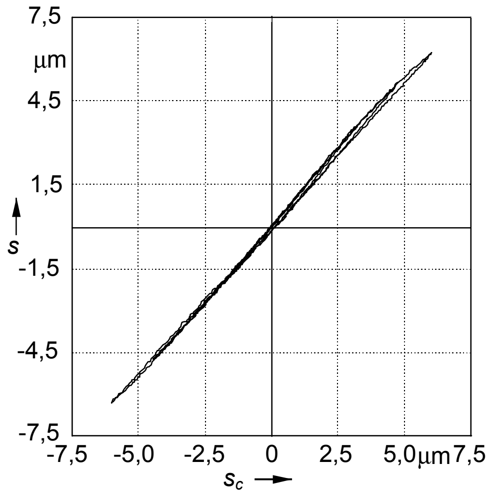

Figure 8: Compensated hysteretic displacement - given displacement relation

As a final result figure 8 shows the compensated characteristic of the overall system given by the serial combination of the inverse feedforward controller and the magnetostrictive actuator. In this example the control error defined by

$$\frac{\max\limits_{t_0 \le t \le t_e} \left\{ \left| H^{-1}[H[s_c]](t) - s(t) \right| \right\}}{\max\limits_{t_0 \le t \le t_e} \left\{ \left| H^{-1}[H[s_c]](t) \right| \right\}} \tag{21}$$

will be strongly reduced to about 3 % due to the inverse feedforward control strategy.

CONCLUSIONS

The main contribution of this paper is to extend the Prandtl-Ishlinskii modeling approach for complex hysteretic nonlinearities to a robust compensator design method for invertible complex hysteretic nonlinearities of the Prandtl-Ishlinskii type. For this purpose the threshold-discrete version of the Prandtl-Ishlinskii hysteresis operator was formulated with linear inequality constraints for the model parameters which guarantee the invertability of the model. Based on these linear inequality constraints and an error model which is linear dependent on the model parameters the identification problem can be formulated as a quadratic program which provides always the best invertable L_2^2-norm approximation of the measured output-input data. The corresponding compensator can be directly calculated and thus efficiently implemented from the model by analytical transformation laws. Finally the compensator design method is used to generate an inverse feedforward controller for a magnetostrictive actuator. In comparision to the conventional controlled magnetostrictive actuator the nonlinearity error of the inverse controlled magnetostrictive actuator is lowered from about 30 % to about 3 %. In future works the method will be extended to hysteresis operators which are also able to model complex hysteretic nonlinearities with asymmetrical hysteretic loops. These type of nonlinearities occurs if magnetostrictive or piezoelectric actuators are driven with higher input amplitudes.

ACKNOWLEDGEMENTS

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