AN OPERATOR-BASED CONTROLLER CONCEPT FOR SMART PIEZO-ELECTRIC STACK ACTUATORS

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1. Introduction

Solid-state actuators based on piezoelectric materials are characterized by forces reaching the kilonewton range and reaction times on the order of microseconds. However, the displacements are low because the maximum strain amounts to 1.5 .. 2 ‰. If they are used in micropositioning drives, the piezoelectric stack actuators are mainly driven with high voltages $v(t)$ to achieve the largest possible displacements $s(t)$. In large-signal operation the actuator characteristic shows strong hysteresis and creep effects which can be regarded as undesired internal disturbances for the micropositioning process. Normally piezoelectric actuators are subsystems in an overlying mechanical structure. During the micropositioning process the mechanical structure reacts with a force $f(t)$ against the piezoelectric transducer. This reaction force has a strong influence on the displacement and can therefore be regarded as an undesired external disturbance for the micropositioning process. Under normal operating conditions the displacement of the piezoelectric stack transducer can be seperated, at least in a first order approximation, into a creep and hysteretic voltage-dependent part described here by a scalar operator Γ_a and a linear force-dependent part characterised by the small-signal elasticity S

$$s(t) = \Gamma_a[v](t) + S \cdot f(t) \ . \tag{1}$$

Equation (1) is the so-called operator-based actuator model of the piezoelectric stack transducer [6].

Additionally, the piezoelectric transducer has an inherent sensory capacity which originates from the same internal microphysical process as the actuator capacity. In sensor operation of the transducer the voltage-dependent part of the electrical charge signal $q(t)$ can be regarded as an external disturbance whereas the force-dependent part of the electrical charge signal contains the measurement information about the force acting on the transducer. Under normal operating conditions the electrical charge signal can be separated, at least in a first order approximation, into a creep and hysteretic voltage-dependent part described here by a scalar operator Γ_e and a linear force-dependent part characterized by the so-called small-signal piezoelectric constant d

$$q(t) = \Gamma_e[v](t) + d \cdot f(t). \tag{2}$$

Equation (2) is the so-called operator-based sensor model of the piezoelectric stack transducer [6].

2. Operator-based Controller Concept

The internal hysteretic and creep disturbances and the external force-generated disturbance lead to ambiguities in the characteristic of piezoelectric actuators and thus to a considerable reduction of the repeatability attainable in open-loop control. A possible solution of this problem is to compensate the hysteresis, creep and force-dependence simultaneously using the inverse feed-forward compensator realized by

$$v(t) = \Gamma_a^{-1}[s_c - S \cdot f](t). \tag{3}$$

In this equation $s_c(t)$ is the desired diplacement. The force $f(t)$ acting on the actuator has to be measured by an external force sensor. Γ_a^{-1} is the inverse operator of Γ_a.

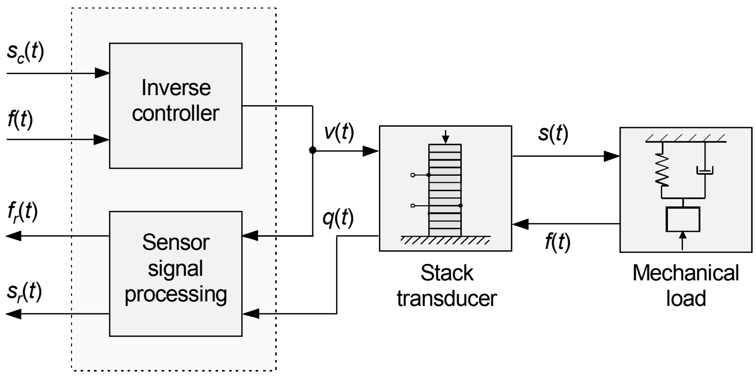


Figure 1. Inverse control of a smart piezoelectric stack actuator

Due to the multi-functional property of the piezoelectric stack transducer the displacement $s(t)$ of the actuator and the force $f(t)$ acting on it can be reconstructed during operation using a signal processing unit, see Figure 1. The signal processing unit bases on the equation

$$f_r(t) = \frac{1}{d} \cdot (q(t) - \Gamma_e[v](t)) \tag{4}$$

to compute the reconstructed force $f_r(t)$ and on the equation

$$s_r(t) = \Gamma_a[v](t) + \frac{S}{d} \cdot (q(t) - \Gamma_e[v](t)), \tag{5}$$

to compute the reconstructed displacement $s_r(t)$ of the transducer. Solid-state transducers which are used simultaneously as both actuators and sensors are frequently called smart actuators. In addition to the inverse controller the controller concept shown in Figure 1 contains a signal processing unit in order to realize such a smart piezoelectric stack actuator for the large-signal operating range. The basis of the feed-forward controller and the signal processing unit is comprised of so-called elementary creep and hysteresis operators which are mathematically simple and which reflect the qualitative properties of the transfer characteristic of the transducer.

3. Operator-based Hysteresis and Creep Modelling

In the mathematical literature the notation of 'hysteretic nonlinearity' will be equated with the notation 'rate independent memory effect' [1]. This means that the output signal of a system with hysteresis depends not only on the present value of the input signal but also on the order of their amplitudes, especially their extremum values, but not on their rate in the past. Because of its phenomenological character the concept of hysteresis operators developed by Krasnosel'skii and Pokrovskii in the 1970's allows a very general and precise modelling of hysteretic system characteristics [3].

The basic idea consists of the modelling of the real hysteretic transfer characteristic by the weighted superposition of many elementary hysteresis operators, which differ in terms of one or more parameters depending on the type of the elementary operator. One type of such an elementary hysteresis operator is the so-called play operator

$$z_r(t) = H_r[v, z_{r0}](t) \tag{6}$$

which is defined by the recursive equations

$$z_r(t) = \max\{v(t) - r, \min\{v(t) + r, z_r(t_i)\}\} \tag{7}$$

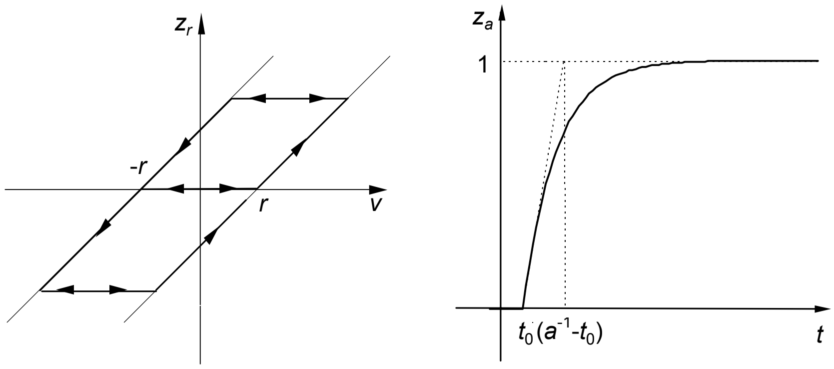
with

$$z_r(t_0) = \max\{v(t_0) - r, \min\{v(t_0) + r, z_{r0}\}\} \tag{8}$$

for piecewise monotonous input signals with a monotonicity partition $t_0 \leq t_1 \leq . . \leq t_i \leq t \leq t_{i+1} . . \leq t_N$. The operator is characterized by its threshhold parameter r, $z_r(t)$ is the operator output and z_{r0} its initial value. Figure 2a shows the rate-independent output-input trajectory of this simple hysteresis operator.

For the precise modelling of real hysteresis phenomena n play operators with different threshold values r_i can be multiplicated with weights b_i and then superimposed. This parallel connection of elementary hysteresis operators leads to the complex hysteresis operator

$$H[v](t) = \sum_{i=1}^{n} b_i \cdot H_{r_i}[v, z_{r_i 0}](t). \tag{9}$$



a) b)

Figure 2. a) Rate-independent transfer characteristic of the play operator
b) Step response of a rate-dependent, elementary creep operator

The notion of creep originates from the field of solid-mechanics and describes the time-variant deformation behaviour of a body due to a sudden mechanical load [2]. It is a strongly damped, rate-dependent phenomenon, which can be found in a similar form in the field of ferromagnetism and ferroelectricity. Like hysteresis phenomena electrically induced creep effects have a considerable influence on the large-signal transfer characteristic of a piezoelectric transducer [5]. In a first order approximation this creep phenomena is presumed linear. As a consequence it can be described, analogously to the hysteresis modelling process, by a complex linear creep operator

$$L[v](t) = \sum_{j=1}^{m} c_j \cdot L_{a_j}[v, z_{a_j 0}](t), \tag{10}$$

given by a weighted superposition of m elementary linear creep operators with different creep eigenvalues a. c_j are the weights of the elementary operators. In this case the elementary linear creep operator is the solution operator

$$L_a[v, z_{a0}](t) = \mathrm{e}^{-a(t-t_0)} \cdot z_{a0} + a \cdot \int_{t_0}^{t} \mathrm{e}^{a(\tau - t)} \cdot v(\tau)\, \mathrm{d}\tau \tag{11}$$

of a linear, first-order differential equation with an initial value z_{a0}. Figure 2b shows the step response of the elementary linear creep operator, which has the same qualitative features as the step response of the creep phenomena in the real system.

In the following we derive a first-order approximation model of the actuator characteristic Γ_a and the electrical characteristic Γ_e by the linear superposition of a weighted reversible part, a rate-independent irreversible part described by the complex hysteresis operator H and a rate-dependent part described by the complex linear creep operator L. From this follows

$$\Gamma_a[v](t) = d \cdot v(t) + H_a[v](t) + L_a[v](t) \tag{12}$$

and

$$\Gamma_e[v](t) = C \cdot v(t) + H_e[v](t) + L_e[v](t). \tag{13}$$

In electrical small signal operation (12) and (13) can be reduced to the reversible part with the piezoelectric constant *d* and the small-signal capacity *C*. From this follows that the operator-based approach is a logical extension of small-signal modelling to the large-signal range.

The problem to find the inverse control value $v(t)$ for a given control value $s_c(t)$ and thus to get the inverse operator Γ_a^{-1} is equivalent to the solution of the operator equation

$$v(t) = P_a^{-1}[s_c - L_a[v]](t) \tag{14}$$

with the hysteresis operator

$$P_a[v](t) := d \cdot v(t) + H_a[v](t). \tag{15}$$

The solution of the operator equation (14) and thus the inverse operator Γ_a^{-1} exists and is unique under the natural inequality constraints

$$0 < d < \infty \,, \quad 0 \le b_i < \infty \;\;;\;\; i = 1\,..\,n, \tag{16}$$

and

$$0 \le c_j < \infty \;\;;\;\; j = 1\,..\,m \,, \quad 0 < a_j < \infty \;\;;\;\; j = 1\,..\,m \tag{17}$$

[4]. An efficient numerical procedure for the solution of the operator equation (14) in real-time is also given in [4].

4. Results and Discussion

The inverse controller for the compensation of hysteresis, creep and force-dependence of the piezoelectric transducer and the measurement signal processing for the reconstruction of the displacement of the transducer and the force on it was realized on a digital signal processor (DSP) with a sampling rate up to 10 kHz.

To verify the performance of the compensation concept the inverse compensator was driven with the desired displacement signal $s_c(t)$ shown in Figure 3a. Figure 3c shows the characteristic of the conventional linear controller as a gray line. It is an ideal linear rate-independent characteristic typical for conventional voltage-amplifiers. As a consequence the characteristic of the serial combination conventional controller-transducer, shown in Figure 3d as a gray line, shows the hysteresis and creep effects of the transducer. The characteristic of the operator-based inverse compensator, shown in Figure 3c as a black line, is obviously inverse to the characteristic of the transducer. As a consequence the characteristic of the serial combination inverse compensator-transducer, shown in Figure 3d as a black line, is almost completely free of hysteresis and creep effects and the displacement error caused by creep and hysteresis effects is reduced from 2.47 μm using the conventional controller to 0.25 μm using the inverse compensator. This is an improvement of one order of magnitude.

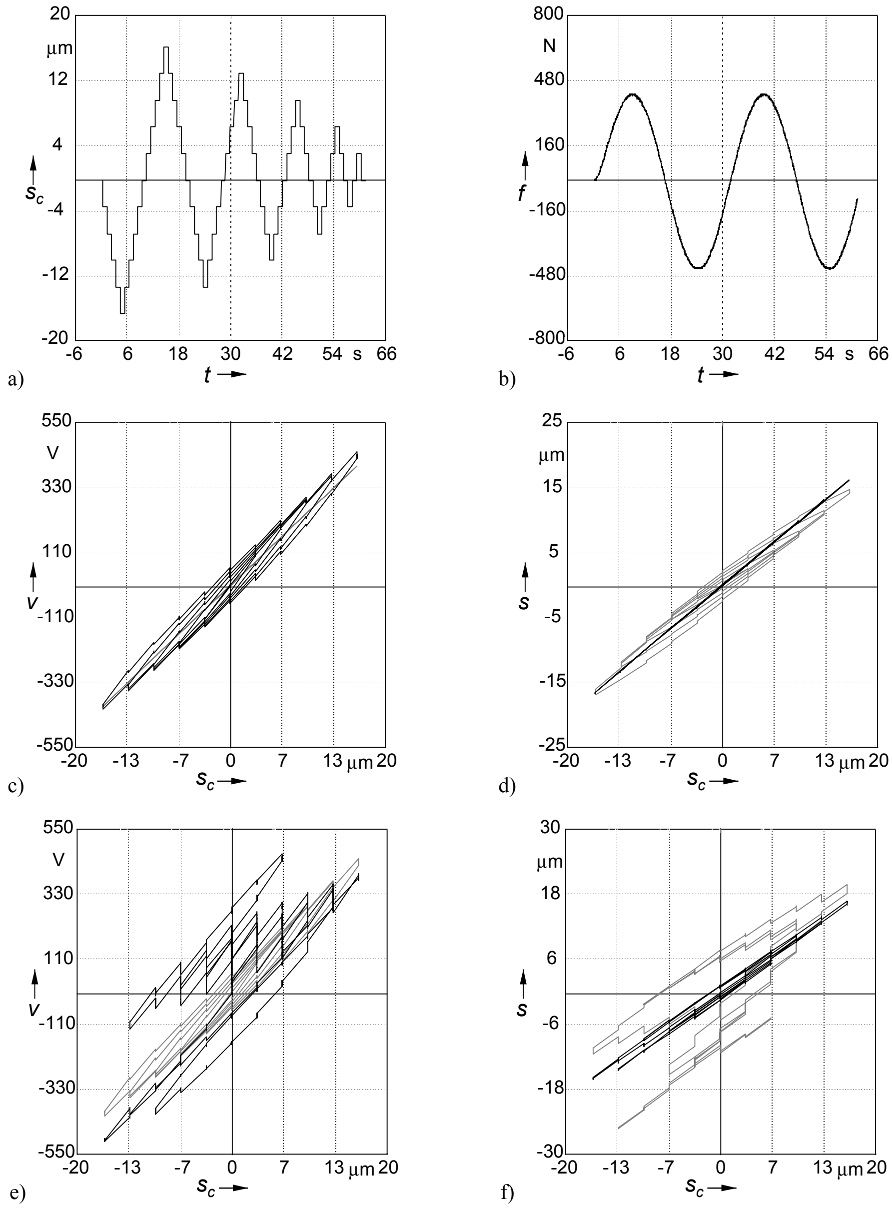


Figure 3. Hysteresis, creep and force-dependence compensation results

The gray line in Figure 3f shows the strongly disturbed characteristic of the serial combination inverse compensator-transducer. The disturbance is generated by the additional external force signal shown in Figure 3b. In this case the force-dependent part of the displacement is not compensated by the inverse controller, see the gray line in Figure 3e. This force-dependence effect leads to a displacement error of 11.8 µm. The black line in Figure 3e shows the strongly disturbed characteristic of the inverse controller. The disturbance of the inverse hysteretic and creep characteristic is caused by an additional compensation of displacement generated by the external force signal. As a consequence the force generated disturbances in the characteristic of the serial combination inverse compensator-transducer is strongly reduced, see the black line in Figure 3f. As the main result the displacement error caused by the external force is reduced to 1.55 µm. This is also an improvement of nearly one order of magnitude.

To verify the performance of the reconstruction concept the transducer was driven with the measured voltage signal $v(t)$ shown in Figure 4a. At the same time the force signal $f(t)$ shown in Figure 4c as a black curve acts on the transducer. The voltage- und force-generated displacement and charge signal of the transducer, $s(t)$ resp. $q(t)$, are shown in Figure 4d and Figure 4b as black curves.

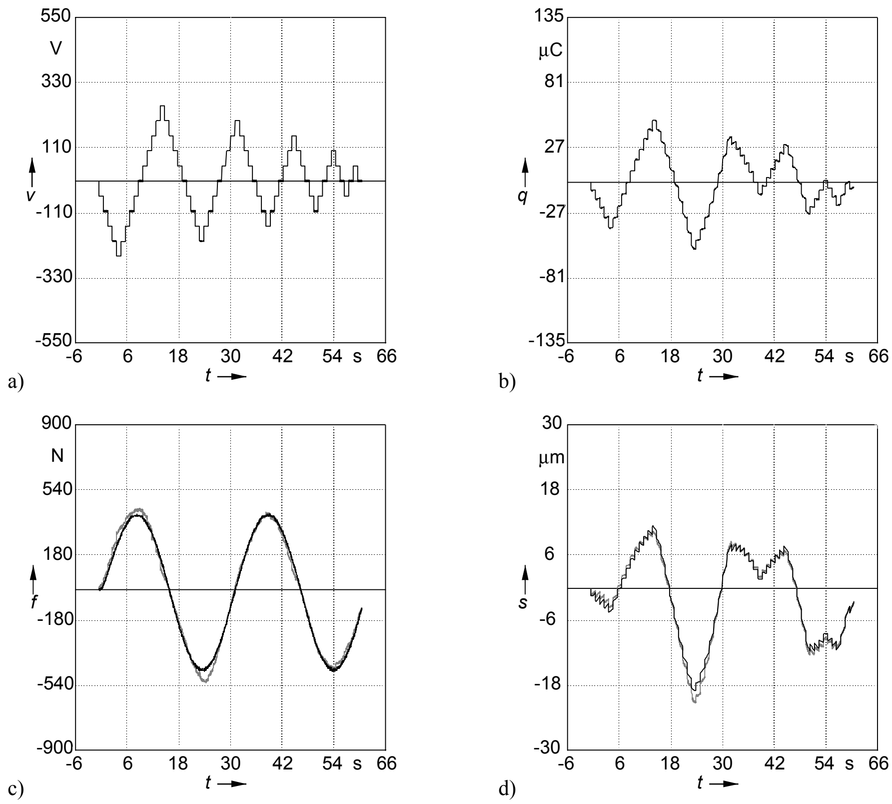


Figure 4. Reconstruction results

The gray curve in Figure 4c is the force signal reconstructed by operator equation (4). The maximum value of the relative deviation between the real force signal and the reconstructed force signal amounts to 15 %. The gray curve in Figure 4d shows the displacement signal reconstructed by operator equation (5). The maximum value of the relative deviation between the real displacement signal and the reconstructed displacement signal amounts to 10 %. If we neglect the hysteresis and creep operators in (12) and (13) we get simplified linear reconstruction models for (4) and (5) which follow directly from the small-signal constitutive relations of the piezoelectric transducer. But due to the hysteresis and creep operators in (12) and (13) the deviation between the measured and calculated voltage-dependent part of the charge signal and the displacement signal in (4) an (5) is reduced by about one order of magnitude [6]. Therefore the reconstruction error of the force signal increases up to 150 % and the reconstruction error of the displacement signal increases up to 100 % if we use the linear reconstruction modells instead of the operator-based versions.

5. Summary

This paper has shown that complex creep and hysteresis operators offer an efficient method to model the hysteresis and creep in the electrical and actuator characteristic of a piezoelectric stack transducer if it is driven electrically in the large-signal range. Based on this method an inverse controller for the simultaneous compensation of hysteresis and creep effects in real-time and a sensor signal processing unit to reconstruct the displacement of the transducer and the force acting on it was presented. In addition to hysteresis and creep phenomena which can be regarded as intrinsic disturbances, the influence of the force on the displacement signal and the control voltage on the charge signal, the external disturbances of the system, are also considered by the new controller concept.

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